CAVITY BASICS

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Outline

- Maxwell equations
- Guided propagation
- Rectangular waveguide
- Circular waveguide
- Vacuum pumping ports
- Directional Couplers
- Pulse compressor
- Resonant cavity
- RF parameters for a resonant cavity
- RF cavities with Superfish

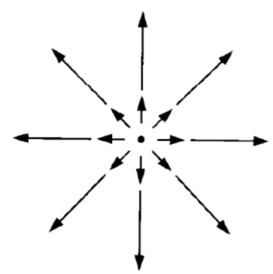
Maxwell Equations[1]

Few mathematics…

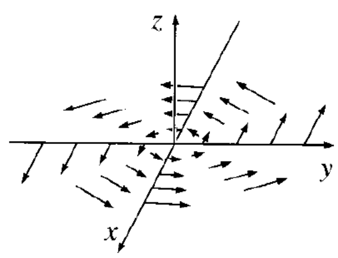
Differential Operators:

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$\nabla \cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$



$$\nabla \times A = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z}$$



Maxwell Equations

I. $\nabla \times E = -\frac{\partial B}{\partial t}$

II. $\nabla \times H = \frac{\partial D}{\partial t} + j$

III. $\nabla \cdot D = \rho$

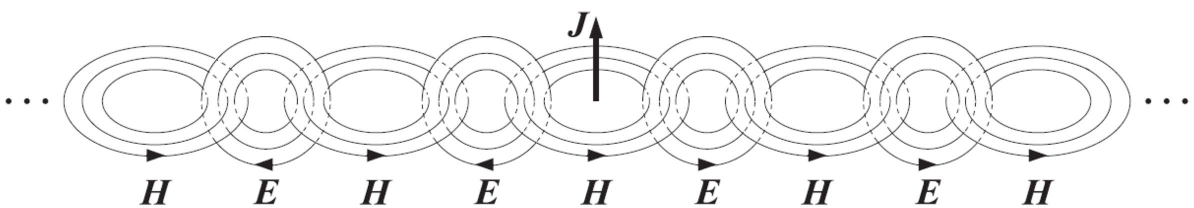
IV. $\nabla \cdot B = 0$

***E** [volt/m] and **H** [ampere/m] are the electric and magnetic field intensities.*

***D** [coulomb/m^2] and **B** [tesla] are the electric and magnetic flux densities.*

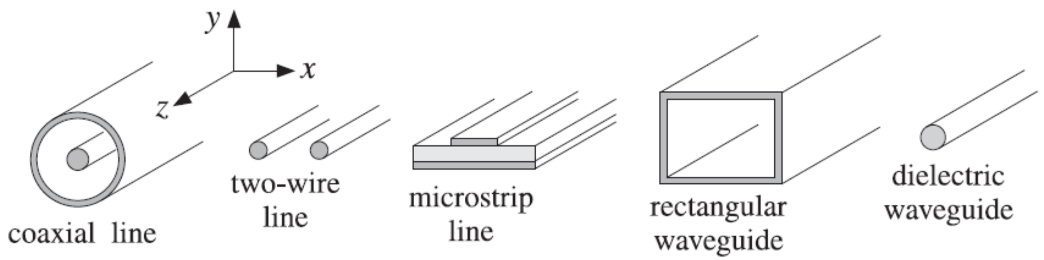
*ρ [coulomb/m^3] and **J** [ampere/m^2] are the volume charge density and electric current density of any external charges.*

A qualitative example



A time-varying current J on a linear antenna generates (i.e. II Maxwell Eq.) a circulating and time varying magnetic field which generates (i.e. I Maxwell Eq.) a circulating magnetic field. Again, the above mentioned electric field generates (i.e. II Maxwell Eq.) a magnetic field, and so on. The cross-linked electric and magnetic field propagate away from the current source.

Guided Propagation[1]



In a waveguiding system, we look for solutions of Maxwell's equations that are propagating along the guiding direction (the z direction) and are confined in the near vicinity of the guiding structure. Thus, the electric and magnetic fields are assumed to have the form:

$$\boxed{\begin{aligned} E(x,y,z,t) &= E(x,y)e^{j(\omega t-\beta z)} \\ H(x,y,z,t) &= H(x,y)e^{j(\omega t-\beta z)} \end{aligned}}$$

where β is the propagation wavenumber along the guide direction. The corresponding wavelength, called *guide wavelength*, is given by $\lambda_g = 2\pi/\beta$.

Guided Propagation

The precise relationship between ω and β depends on the type of waveguide structure and the particular propagating mode.
Because of the preferential role played by the guiding direction z, it is convenient to decompose Maxwell's equations into longitudinal and transverse components.

Thus, we decompose:

$$E(x,y) = \underbrace{E_x(x,y)\hat{x} + E_y(x,y)\hat{y}}_{\textbf{transverse}} + \underbrace{E_z(x,y)\hat{z}}_{\textbf{longitudinal}} = E_T(x,y) + E_z(x,y)\hat{z}$$

In a similar way we can decompose the gradient operator:

$$\nabla = \underbrace{\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y}}_{\textbf{transverse}} + \frac{\partial}{\partial z}\hat{z} = \nabla_T - j\beta\hat{z}$$

Guided Propagation

Depending on whether both, one or none of the longitudinal components are zero, solutions are classified as transverse electric and magnetic (TEM), transverse electric (TE), transverse magnetic (TM), or hybrid:

$$\boxed{\begin{array}{ll} E_z = 0 \; ; H_z = 0 & \text{TEM modes} \\ E_z = 0 \; ; H_z \neq 0 & \text{TE modes} \\ E_z \neq 0 \; ; H_z = 0 & \text{TM modes} \\ E_z \neq 0 \; ; H_z \neq 0 & \text{hybrid modes} \end{array}}$$

Guided Propagation

In case of guided propagation it is also defined the so-called *cutoff wavenumber* k_c given by:

$$\boxed{k_c^2 = \omega^2 \varepsilon\mu - \beta^2 = \frac{\omega^2}{c^2} - \beta^2 = k^2 - \beta^2}$$ (cutoff wavenumber)

The quantity $k = \omega/c = \omega/\sqrt{\epsilon\mu}$ is the wavenumber a uniform plane wave would have in the propagation medium ε, μ.
Although k_c^2 stands for the difference $\omega^2\varepsilon\mu - \beta^2$, it turns out that the boundary conditions for each waveguide type force k_c^2 to take on certain values, which can be positive, negative, or zero, and characterize the propagating modes.
Also we introduce the cuttoff frequency and the cutoff wavelength:

$$\boxed{\omega_c = ck_c \quad \lambda_c = \frac{2\pi}{k_c}}$$ (cutoff frequency & cutoff wavelength)

Guided Propagation

Introducing the longitudinal-transverse decomposition in Maxwell's equations we can write the following set of equations:

$$\boxed{\begin{aligned} \nabla_T^2 E_z + k_c^2 E_z &= 0 \\ \nabla_T^2 H_z + k_c^2 H_z &= 0 \end{aligned}}$$

(Helmholtz Equation)

These equations are to be solved subject to the appropriate boundary conditions for each waveguide type. Once, the fields E_z, H_z are known, the transverse fields E_T, H_T can be computed from:

$$\boxed{\begin{aligned} H_T - \frac{1}{\eta_{TM}} \hat{z} \times E_T &= \frac{j}{\beta} \nabla_T H_z \\ E_T - \eta_{TE} H_T \times \hat{z} &= \frac{j}{\beta} \nabla_T E_z \end{aligned}}$$

$$\eta_{TE} = \omega\mu/\beta$$
$$\eta_{TM} = \beta/\omega\varepsilon.$$

This results in a complete solution of Maxwell's equations for the guiding structure. To get the full x, y, z, t dependence of the propagating fields, the above solutions must be multiplied by the factor $e^{j(\omega t - \beta z)}$

Guided Propagation

Operating Bandwidth

All waveguide systems are operated in a frequency range that ensures that only the lowest mode can propagate.
A mode with cutoff frequency ω_c will propagate only if its frequency is $\omega \geq \omega_c$. If $\omega \leq \omega_c$, the wave will attenuate exponentially along the guide direction.

$$k_c^2 = \omega^2 \varepsilon\mu - \beta^2 \quad \Rightarrow \quad \beta^2 = \frac{\omega^2 - \omega_c^2}{c^2}$$

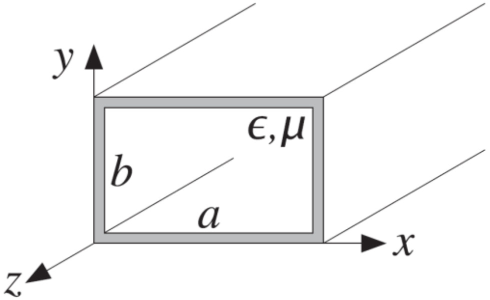
If $\omega \geq \omega_c$, the wavenumber β is real-valued and the wave will propagate. But if $\omega \leq \omega_c$, β becomes imaginary, say, $\beta = -j\alpha$, and the wave will attenuate in the z direction, with a penetration depth $\delta = 1/\alpha$:

$$e^{-j\beta z} = e^{-\alpha z}$$

If we arrange the cutoff frequencies in increasing order, $\omega_{c1} \leq \omega_{c2} \leq \ \omega_{c3}$ ..., then, to ensure single-mode operation, the frequency must be restricted to the interval $\omega_{c1} \leq \omega \leq \omega_{c2}$, so that only the lowest mode will propagate. This interval defines the *operating bandwidth* of the guide.

Rectangular Waveguide

TE_{10} and TE_{n0} Modes



For transverse electric (TE) modes the longitudinal component of the electric field is 0.

$$E_z(x, y) = 0$$

The simplest and dominant propagation mode is the so-called TE_{10} mode and depends only on the x-coordinate. In this case, the Helmholtz equation reduces to:

$$\partial_x^2 H_z(x) + k_c^2 H_z(x) = 0 \quad \Rightarrow \quad H_z(x) = H_0 cos(k_c x)$$

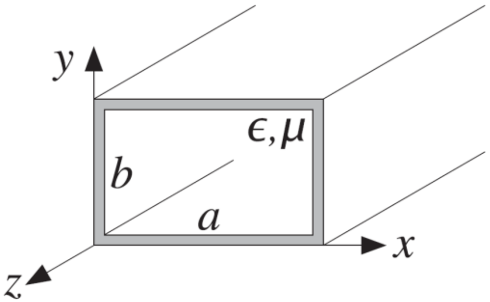
Then, the corresponding electric field will be:

$$E_y(x) = E_0 sin(k_c x)$$

Rectangular Waveguide

TE_{10} and TE_{n0} Modes

$$E_y(x) = E_0 sin(k_c x)$$



Assuming perfectly conducting walls, the boundary conditions require that there be no tangential electric field at any of the wall sides. Because the electric field is in the y-direction, it is normal to the top and bottom sides. But, it is parallel to the left and right sides.

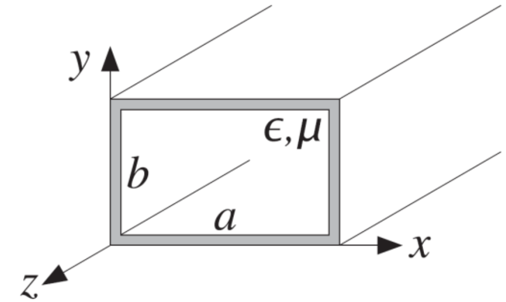
Boundary condition requires that $E_y(a) = 0$, so $k_c a$ must be an integral multiple of π:

$$k_c = \frac{n\pi}{a}$$

Rectangular Waveguide

TE_{10} and TE_{n0} Modes

The corresponding cutoff frequency $\omega_c = ck_c$, $f_c = \omega_c/2\pi$, and wavelength $\lambda_c = 2\pi/k_c = c/f_c$ are:



$$\boxed{\omega_c = \frac{cn\pi}{a} \quad ; \quad f_c = \frac{cn}{2a} \quad ; \quad \lambda_c = \frac{2a}{n}}$$ (TE_{n0} modes)

The dominant mode is the one with the lowest cutoff frequency or the longest cutoff wavelength, that is, the mode TE_{10} having n = 1. It has:

$$\boxed{\omega_c = \frac{c\pi}{a} \quad ; \quad f_c = \frac{c}{2a} \quad ; \quad \lambda_c = 2a}$$ (TE_{10} mode)

Rectangular Waveguide

TE_{nm} and TM_{nm} Modes

To construct higher modes, we look for solutions of the Helmholtz equation that are factorable in their x and y dependence. The most general solutions of Helmholtz equations that will satisfy the boundary conditions are:

$$H_z(x,y) = H_0 cos(k_x x) cos(k_y y) \qquad (TE_{nm} \text{ modes})$$

$$E_z(x,y) = E_0 sin(k_x x) sin(k_y y) \qquad (TM_{nm} \text{ modes})$$

Starting from the previous results we can then derive the transverse components. The boundary conditions require:

$$k_x = \frac{n\pi}{a} \quad ; \quad k_y = \frac{m\pi}{b}$$

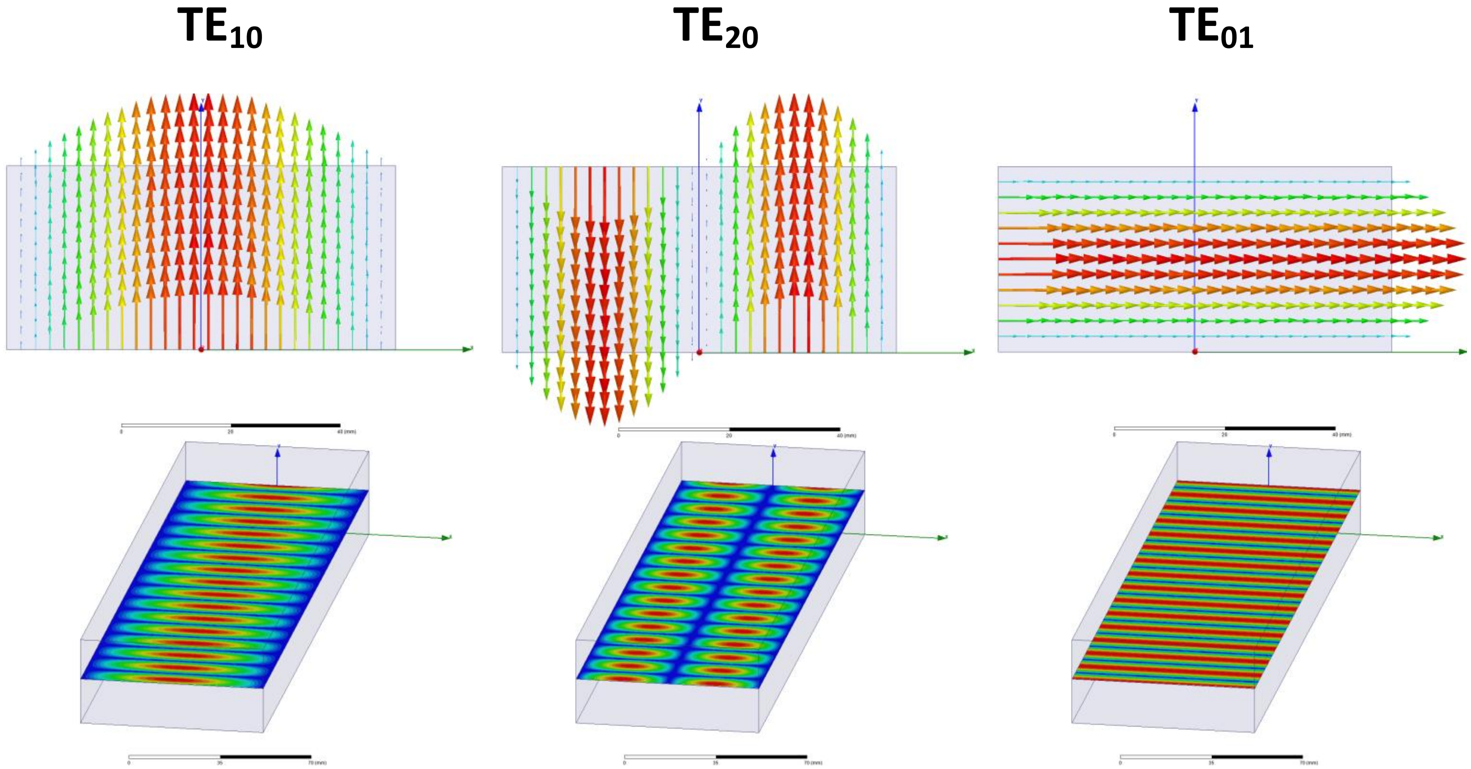
So we obtain:

$$f_{nm} = \sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}$$

Rectangular Waveguide

TE_{nm} and TM_{nm} Modes

Electric field transverse and longitudinal distribution for the first 3 operating modes.



Circular Waveguide

TE_{nm} and TM_{nm} Modes

Like for the rectangular waveguide many modes exist in round waveguides: they are of the transverse electric (TE) and transverse magnetic (TM) type with respect to the axis.

These modes are indexed with two numbers: the first for the azimuthal, the second for the radial 'number of half-waves'.

For TE_{nm} modes

$$k_c = \frac{q_{nm}}{a}$$

For TM_{nm} modes

$$k_c = \frac{p_{nm}}{a}$$

where:

a - is the radius of the circular waveguide

q_{nm} - is the *m*-th zero of the derivatives of Bessel function of order *n*.

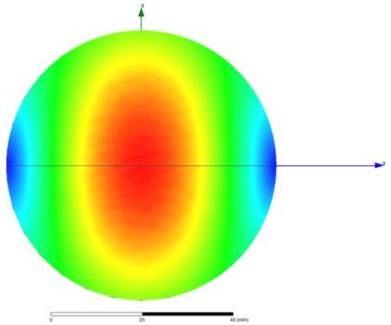
p_{nm} - is the *m*-th zero of Bessel function of order *n*.

Circular Waveguide

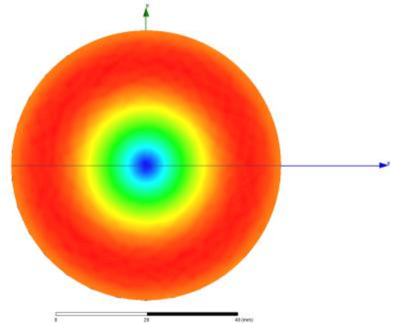
TE_{nm} and TM_{nm} Modes

Electric field transverse distribution for the first 6 operating modes.

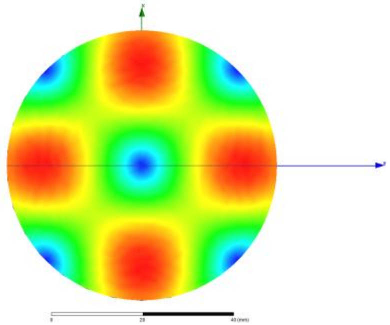
TE_{11}



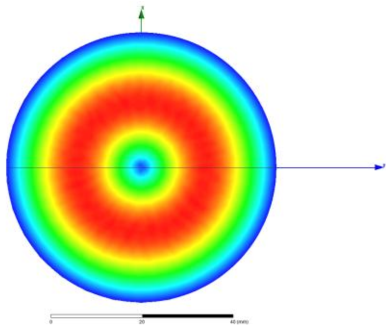
TM_{01}



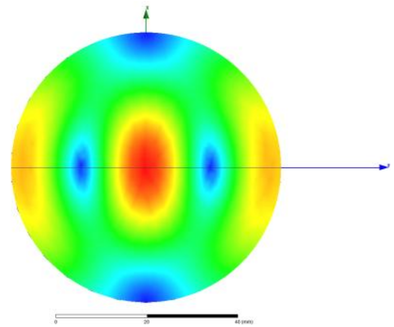
TE_{21}



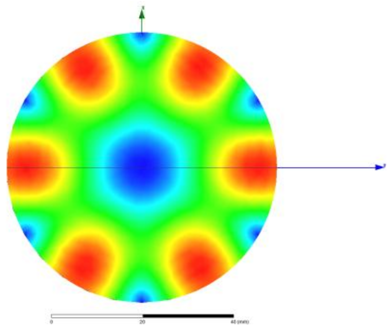
TE_{01}



TM_{11}



TE_{31}

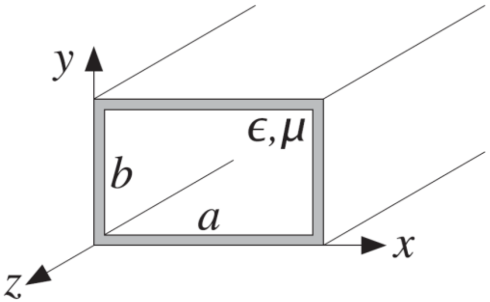


Propagation vs Attenuation

A visual example…

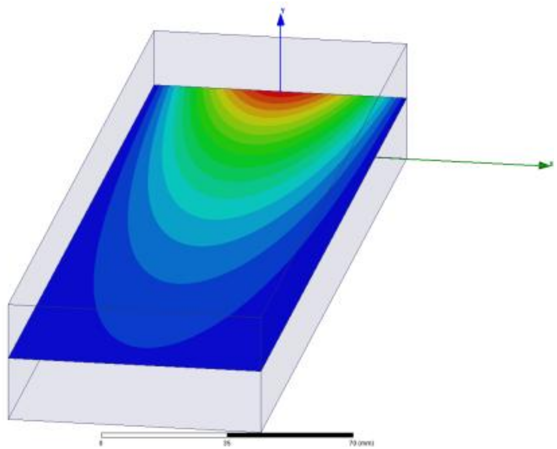
Let us consider a standard rectangular waveguide WR284:

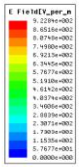
- *a = 72.136 mm*
- *b = 34.036 mm*



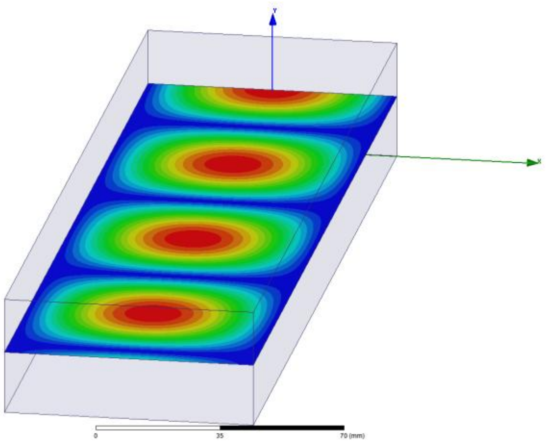
The dominant mode is the TE_{10}. It has: $f_c = c/2a = 2.078\ GHz$

TE_{10} @ 2.0 GHz



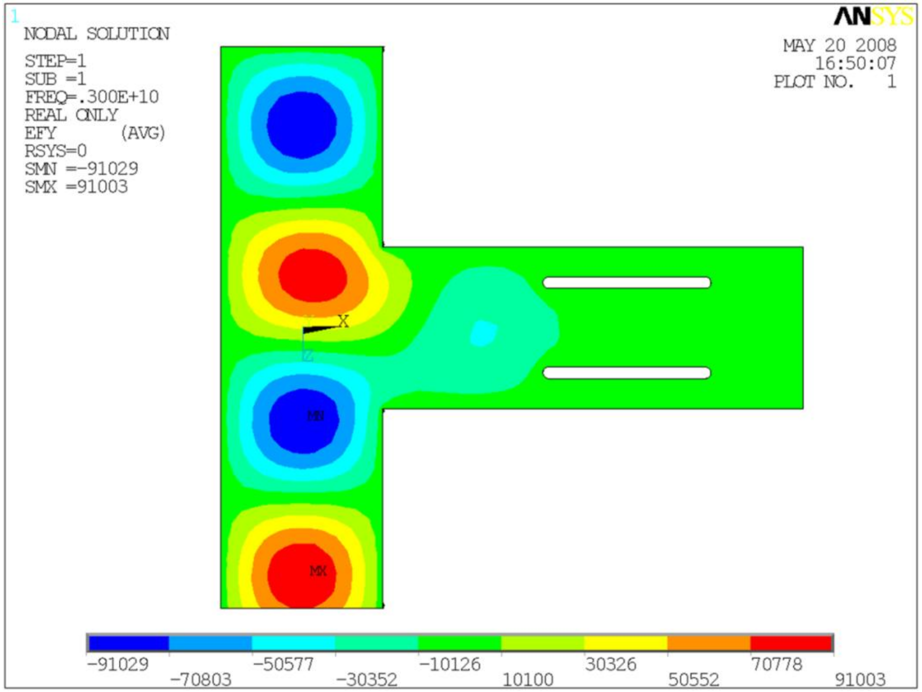


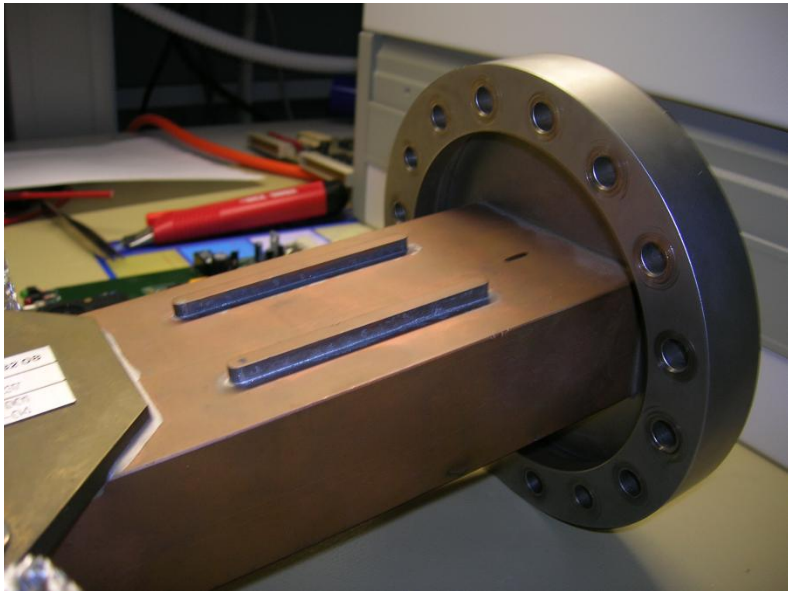
TE_{20} @ 2.998 GHz



Vacuum pumping ports

A WR284 vacuum pumping port

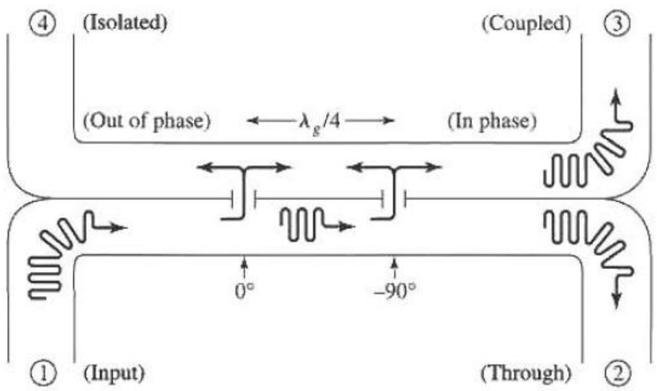




Directional Coupler

Multihole directional couplers

A wave entering at port 1 is mostly transmitted through to port 2, but some power is coupled through the two apertures. If a phase reference is taken at the first aperture, then the phase of the wave incident at the second aperture will be -90'.
Each aperture will radiate a forward wave component and a backward wave component into the upper guide; in general, the forward and backward amplitudes are different.

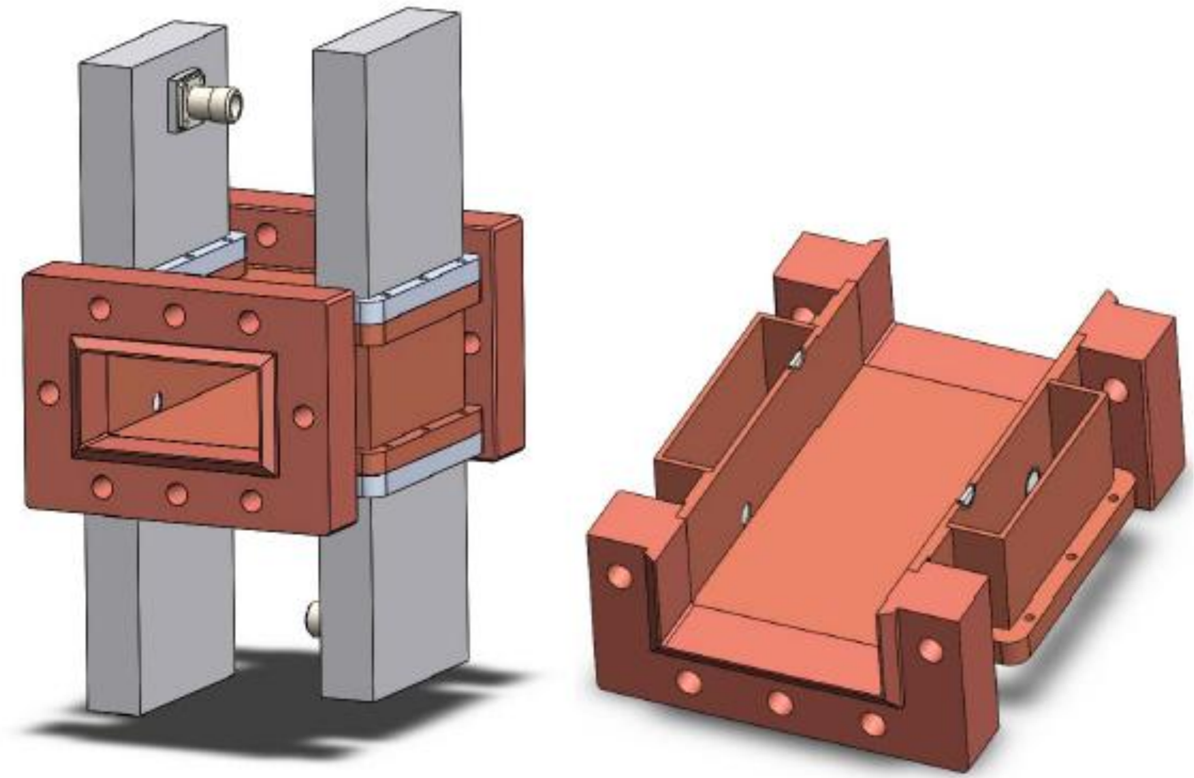


Basic operation of a two-hole directional coupler.

In the direction of port 3, both components are in phase, since both have traveled $\lambda_g/4$ to the second aperture.
But we obtain a cancellation in the direction of port 4, since the wave coming through the second aperture travels $\lambda_g/2$ further than the wave component coming through the first aperture.

Directional Coupler

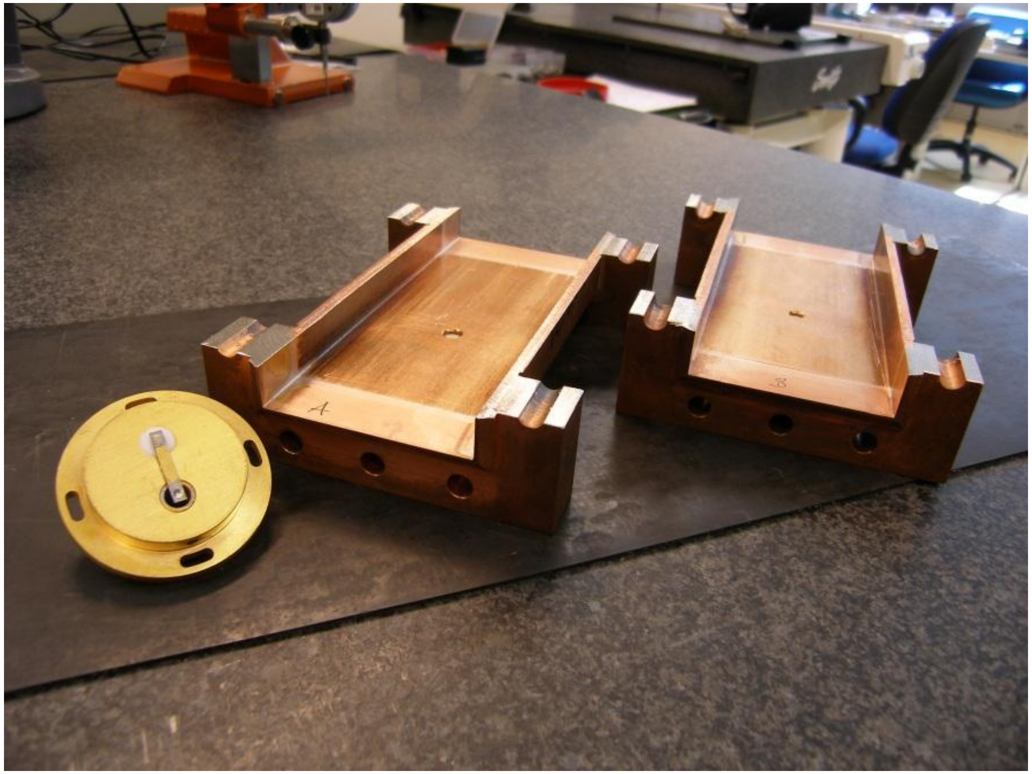
Multihole directional couplers



Courtesy of ***Mega Industries, LLC***

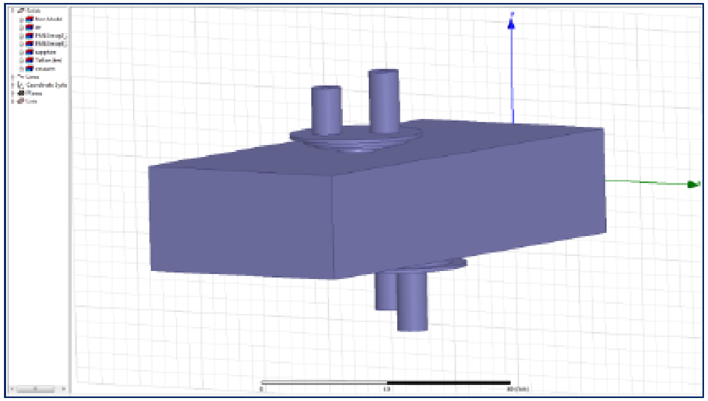
Directional Coupler

Bethe hole directional couplers



Directional Coupler

Bethe hole directional couplers



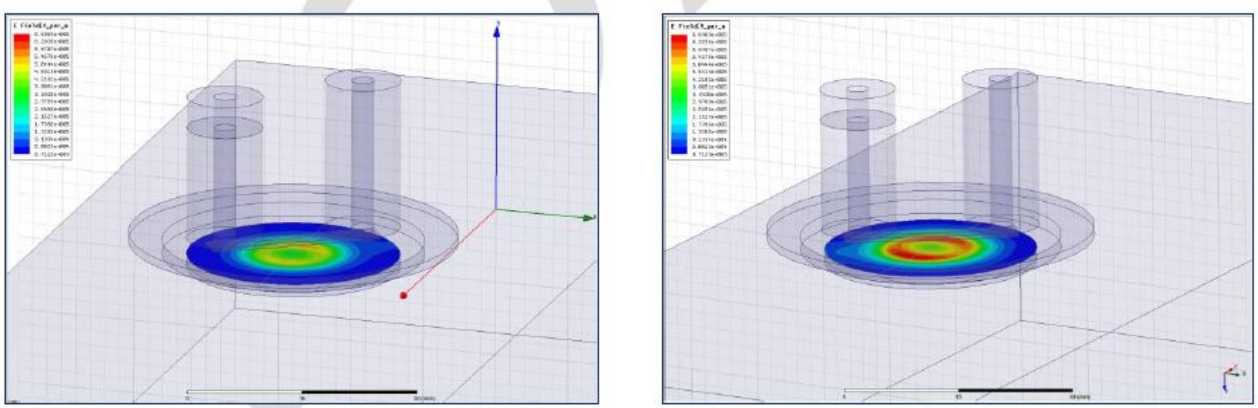
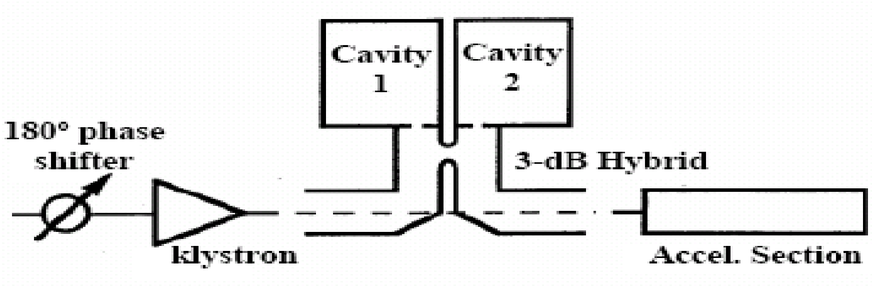
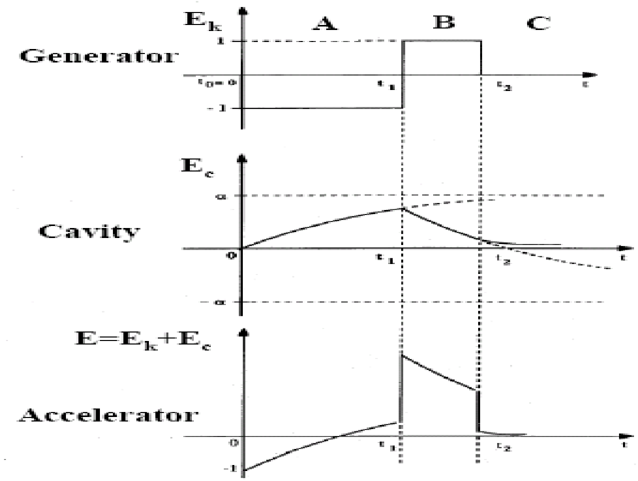


Fig.3 Surface electric field on ceramic surfaces (air side) of the forward wave (left) and backward wave (right) pick up

Pulse Compressor



Sketch of a RF plant equipped with a pulse compressor system

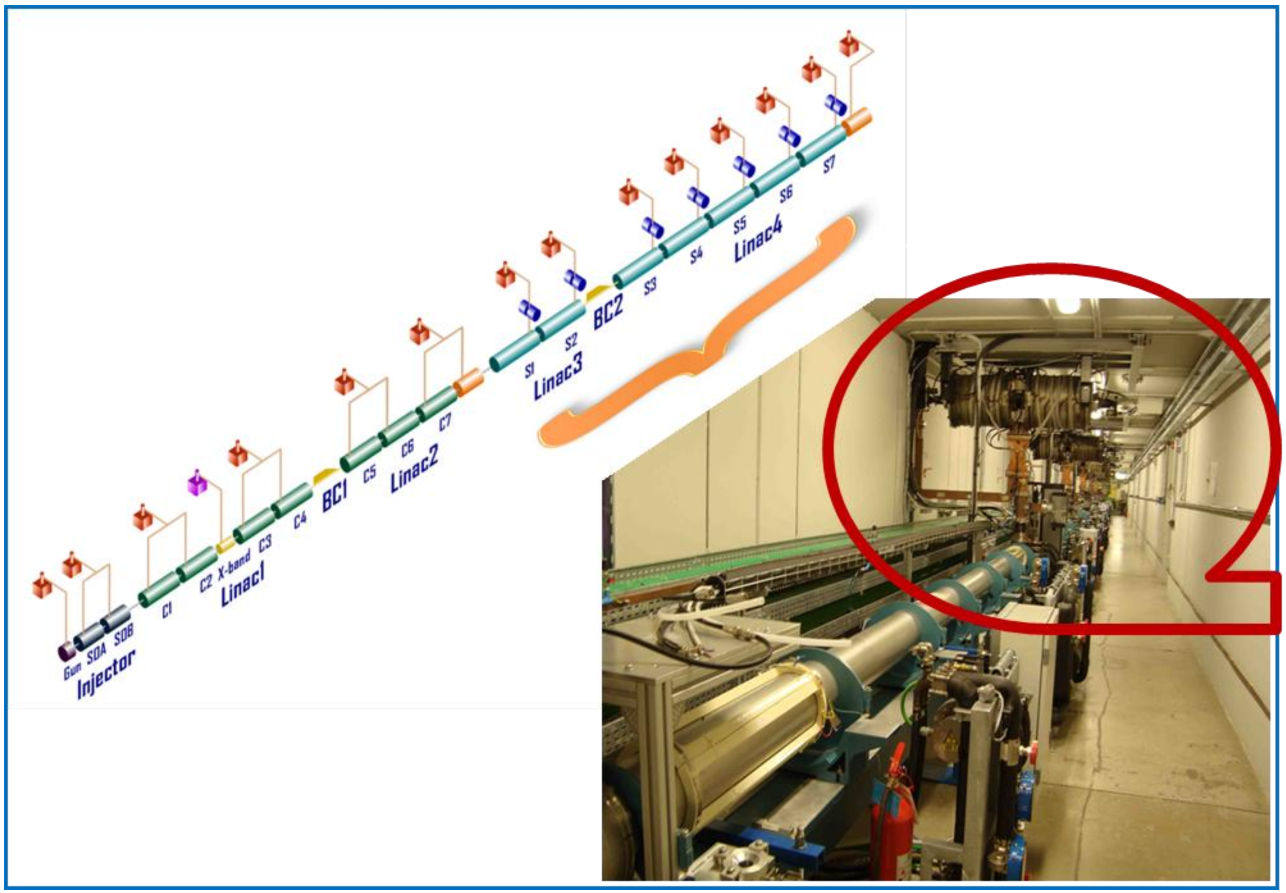


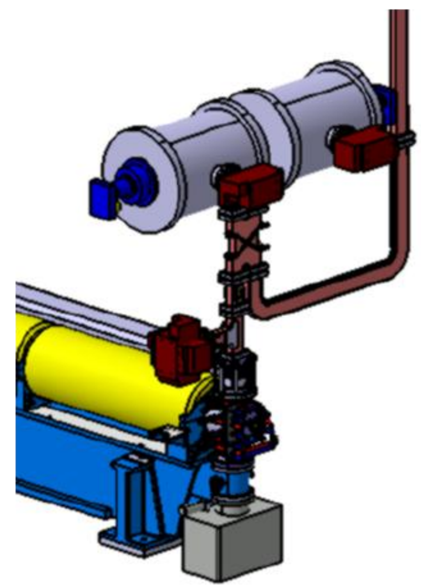
The waveform of conventional SLED

The usual way of operating the SLED system implies that, at a certain instant t_1 (usually t_1 should be more than 2~3 times the *filling time* of the SLED cavities), the phase of the RF wave at the klystron output is reversed by 180º . This effect produces a much higher peak RF power at the input of accelerator, determined by all the parameters, but the waveform is far from being “flat”.

Pulse Compressor

An example…

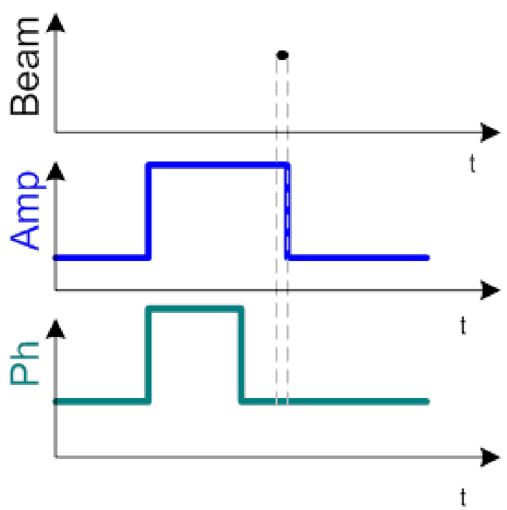


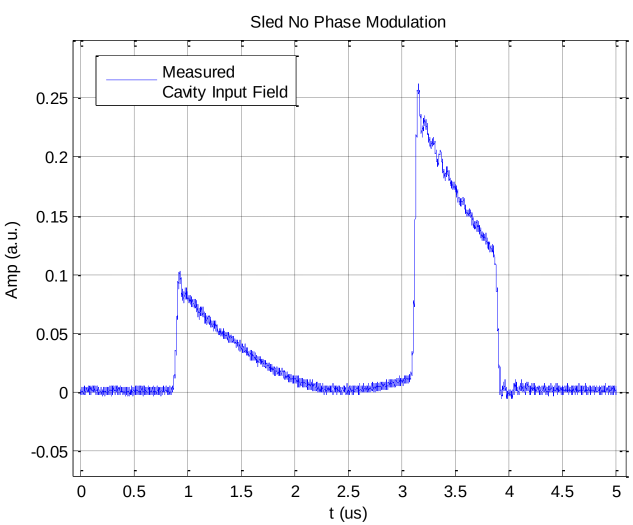


Pulse Compressor

Standard SLED operation at FERMI

- Amplitude of klystron output is kept constant during the RF Pulse
- 180º Phase Shift before the end of the pulse. The pulse duration after the phase reversal must be at least equal to the accelerating structure filling time.





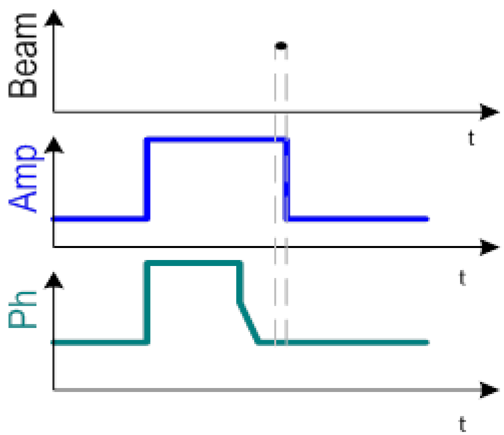
Pulse waveform at FERMI

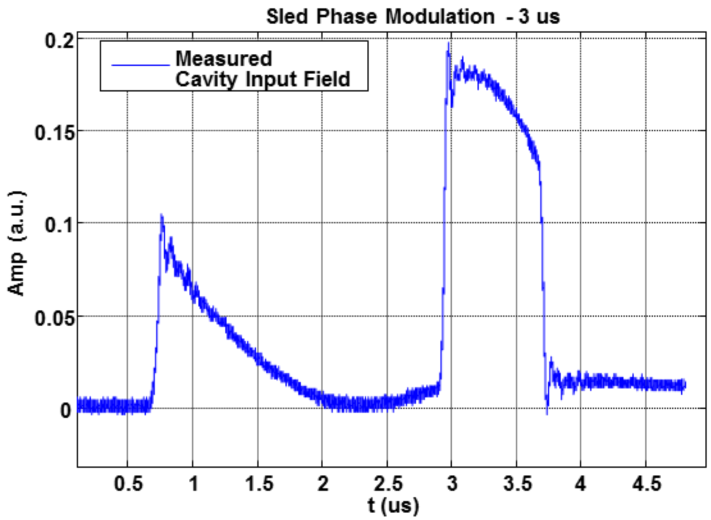
- Measured $E_{pk\text{-}cav}/E_{pk\text{-}kly}$ = 2.5 (3µs pulse)
- Measured Energy Gain Factor = 1.5

Pulse Compressor

Phase Modulation Technique at FERMI

- Amplitude of klystron output is kept constant during the RF Pulse
- 95º Phase Shift 0.77 µs before the end of the pulse
- Phase changed gradually (slope = 149º/µs)
- Phase kept constant during the last 200 ns of the RF pulse





3µs pulse

$E_{pk\text{-}cav}/E_{pk\text{-}kly}$ = 1.83 (field ratio)

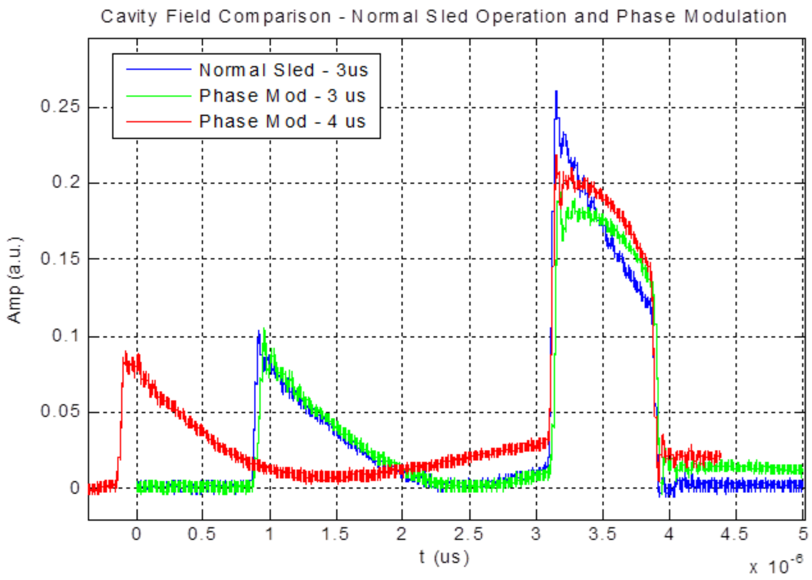
$P_{pk\text{-}cav}/P_{pk\text{-}kly}$ = 3.34 (power ratio)

Note: Phase offset and Phase Slope Parameters adjustable by LLRF

Pulse Compressor

Phase Modulation Technique at FERMI

	RF width [μs]	$E_{pk\text{-}cav}/E_{pk\text{-}kly}$ [field ratio]	Gain Factor
Normal Operation	**3.0**	**2.5**	**1.5**
Phase Modulation	**3.0**	**1.83**	**1.45**
Phase Modulation	4.0	2.1	1.65



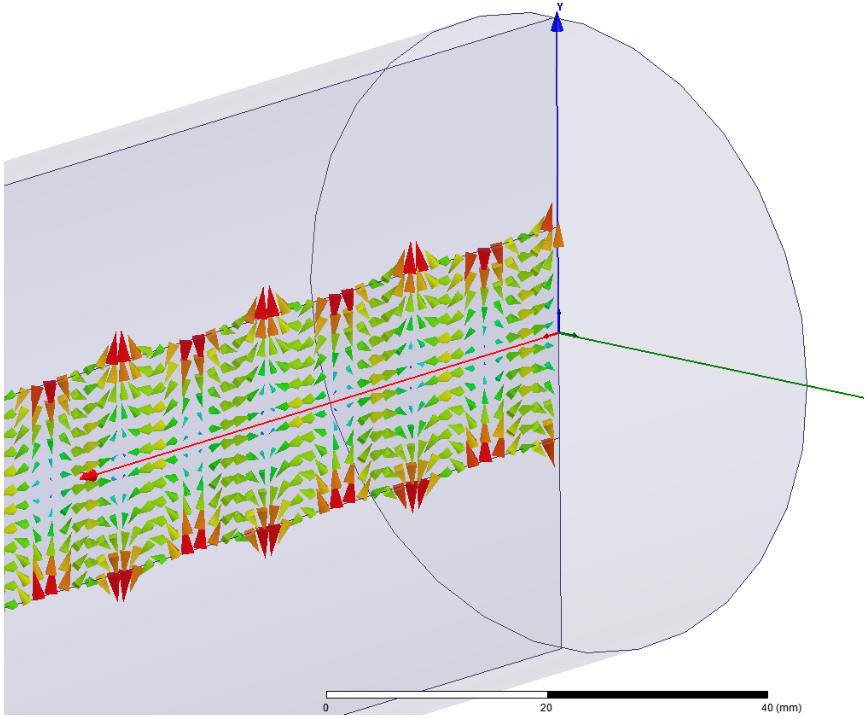
Even if the RF pulse shape is not flat, for a 3 μs overall pulse width the energy gain loss with respect to the normal operation is just 3.3 %

Resonant Cavity for Particle Acceleration

The TM_{01} mode in circular waveguides is of particular interest for accelerating cavities.

This mode is rotational symmetric and has an axial field on axis. A short piece of circular waveguide operated in this mode is in fact a simple form of an accelerating cavity.

(More details will be given in the next lecture)



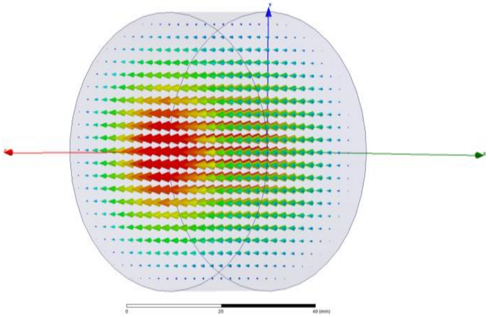
Starting from a round waveguide, and here in particular with the TM_{01} mode, we can construct a cavity simply from a piece of waveguide at its cutoff frequency; this results in the fundamental mode of the so-called pillbox cavity, referred to as TM_{010} mode.

Resonant Cavity for Particle Acceleration

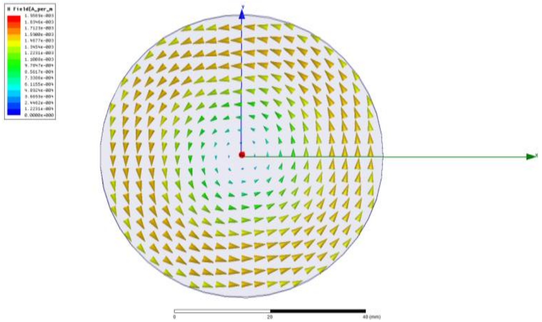
The eigenfrequency of the TM_{010} mode is the cutoff frequency of the waveguide and thus independent of the cavity height h. There is no axial field dependence, indicated by the axial index 0.

The fields of the TM_{010} mode in a simple pillbox cavity (closed at z=0 and z=h and at r=a with perfectly conducting walls) are given by:

TM_{010} Electric Field



TM_{010} Magnetic Field



$$E_z = \frac{1}{j\omega\varepsilon_0}\frac{\chi_{01}}{a}\sqrt{\frac{1}{\pi}}\,\frac{J_0\left(\frac{\chi_{01}}{a}\rho\right)}{aJ_1\left(\frac{\chi_{01}}{a}\right)} \quad ; \quad B_\phi = \mu_0\sqrt{\frac{1}{\pi}}\,\frac{J_0\left(\frac{\chi_{01}}{a}\rho\right)}{aJ_1\left(\frac{\chi_{01}}{a}\right)}$$

where χ_{10} is the first zero of $J_0(x)$, $\chi_{10} = 2.40483$.

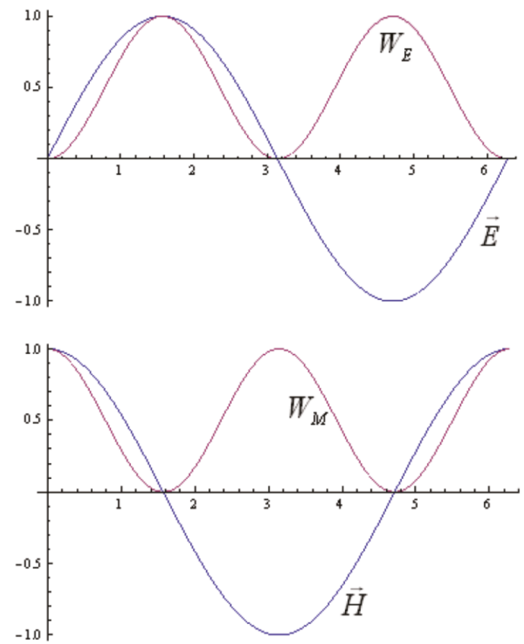
Electric and magnetic fields are out of phase by 90°, as indicated by the *j* in the previous equations.

RF parameters for a resonant cavity

Stored Energy

The total energy stored in a cavity is the sum of the electric field energy and the magnetic field energy.

$$W = \underbrace{\iiint_{cavity} \frac{\varepsilon}{2}\left|\vec{E}\right|^2 dV}_{\textbf{Electric field energy}} + \underbrace{\iiint_{cavity} \frac{\mu}{2}\left|\vec{H}\right|^2 dV}_{\textbf{Magnetic field energy}}$$



The energy is constantly swapping back and forth between these two energy forms at twice the RF frequency; while one is varying in time as $sin^2(\omega t)$, the other one is varying as $cos^2(\omega t)$, such that the sum is constant in time.

Note also that this is related to the fact that $\vec{E}$ and $\vec{H}$ are exactly in quadrature, as we had already seen for the simple pillbox cavity.

RF parameters for a resonant cavity

Quality Factor Q

If the RF cavity would be entirely closed by a perfect conductor and the cavity volume would not contain any lossy material, there would exist solutions to Maxwell's equations with non-vanishing fields even without any excitation.
If, however, the cavity walls are made of a good rather than a perfect conductor, modes' eigenfrequencies will become complex, describing damped oscillations, so each mode will be characterized by its frequency and its decay rate.
If the field amplitudes of a mode decay as $\propto e^{-\alpha t}$, the stored energy decays as $\propto e^{-2\alpha t}$. The quality factor *Q* is defined as:

$$Q = \frac{\omega_0 W}{P_{loss}} = \frac{\omega_0 W}{-\frac{dW}{dt}} = \frac{\omega_0}{2\alpha}$$

Here ω_0 denotes the eigenfrequency and *W* the stored energy. P_{loss} is the power lost into the cavity walls (or any other loss mechanism).

It is clear that the larger the *Q*, the smaller will become the power necessary to compensate for cavity losses.

RF parameters for a resonant cavity

Accelerating Voltage

We define the 'accelerating voltage' of a cavity the integrated change of the kinetic energy of a traversing particle divided by its charge:

$$V_{acc} = \frac{1}{q} \int_{-\infty}^{\infty} q\left(\vec{E} + \vec{v} \times \vec{B}\right) ds$$

where *ds* denotes integration along the particle trajectory. With the fields varying at a single frequency ω and particles moving with the speed βc in the *z* direction, this expression simplifies to:

$$V_{acc} = \int_{-\infty}^{\infty} \vec{E}(z) e^{j\frac{\omega}{\beta c}z} dz$$

In the previous formula we have to consider the field as the complex amplitude of the field of the cavity oscillation mode. The exponential accounts for the movement of the particle with speed βc through the cavity while the fields continue to oscillate.

RF parameters for a resonant cavity

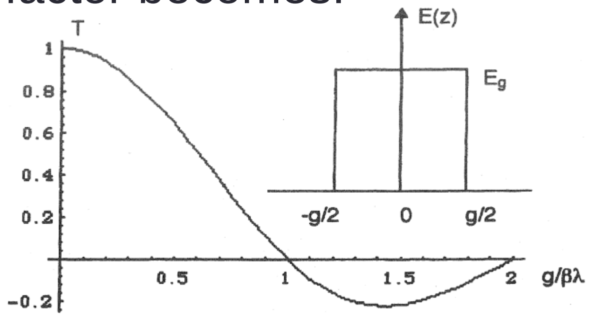
Transit-time factor[3]

In the definition of the accelerating voltage we already accounted for the finite speed of the particles through the cavity. It thus includes already the so-called transit-time factor, which describes this effect alone. The transit-time factor *T* is defined as:

$$T = \frac{|V_{acc}|}{\int_{-\infty}^{\infty} |\vec{E}(z)| dz}$$

For a simple TM_{010} pillbox cavity where the field is constant over a gap of length *g*, and falls to zero outside the gap, the transit-time factor becomes:

$$T = \frac{sin(\pi g/\beta\lambda)}{\pi g/\beta\lambda}$$



To achieve maximum energy gain for a given V_0 we want $T = 1$, which corresponds to $g = 0$. However, other consideration must be taken into account to determine the optimum geometry, such as the risk of RF electric breakdown and RF power efficiency.

RF parameters for a resonant cavity

Shunt Impedance & R/Q

It is convenient to define the *shunt impedance*, a figure of merit that is independent of the excitation level of the cavity and measures the effectiveness of producing an axial voltage for a given power dissipation

$$R = \frac{|V_{acc}|^2}{P_d L} = \frac{|V_0 T|^2}{P_d L} \qquad \left[\frac{M\Omega}{m}\right]$$

(effective shunt impedance per unit length)

Since the energy *W* stored in the cavity is proportional to the square of the field (and thus the square of the accelerating voltage), it can be used to conveniently normalize the accelerating voltage; this leads to the definition of the quantity *R*-upon-*Q*:

$$\frac{R}{Q} = \frac{|V_{acc}|^2}{\omega_0 W}$$

The R/Q quantifies how effectively the cavity converts stored energy into acceleration. Note that R/Q is uniquely determined by the geometry of the cavity.

RF cavities with Superfish[4]

- Superfish is a solver in a collection of programs from LANL for calculating radiofrequency electromagnetic fields in either 2-D Cartesian coordinates or axially symmetric cylindrical coordinates.
- Finite Element Method
- Poisson Superfish is available at the following link: http://laacg.lanl.gov/laacg/services/download_sf.phtml

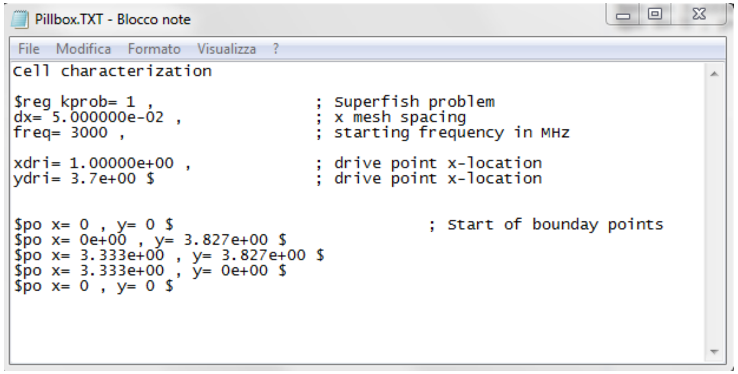
Solvers:

- **Automesh** – generates the mesh (always the first program to run)
- **Fish** – RF solver
- **Cfish** – version of Fish that uses complex variables for the rf fields, permittivity, and permeability.
- **SFO, SF7** – postprocessing
- **Autofish** – combines Automesh, Fish and SFO
- **DTLfish, DTLCells, CCLfish, CCLcells, CDTfish, ELLfish, ELLCAV, MDTfish, RFQfish, SCCfish** – for tuning specific cavity types.
- **Kilpat, Force, WSFPlot**, etc.

A pillbox cavity with Superfish

Superfish input file:

- The input file is a simple .txt file
- The first part of the file is used to:
  a. Define the type of the problem
  b. Define the mesh size (i.e. the size of grid's elements)
  c. Define the starting frequency
  d. Define the position of the 'drive point'
- The second part of the file defines the cell geometry
  a. The geometry is defined point by point
  b. All the dimensions must be in centimiters
  c. The starting point is always x=0 , y=0
  d. The ending point bust be equal to the starting point

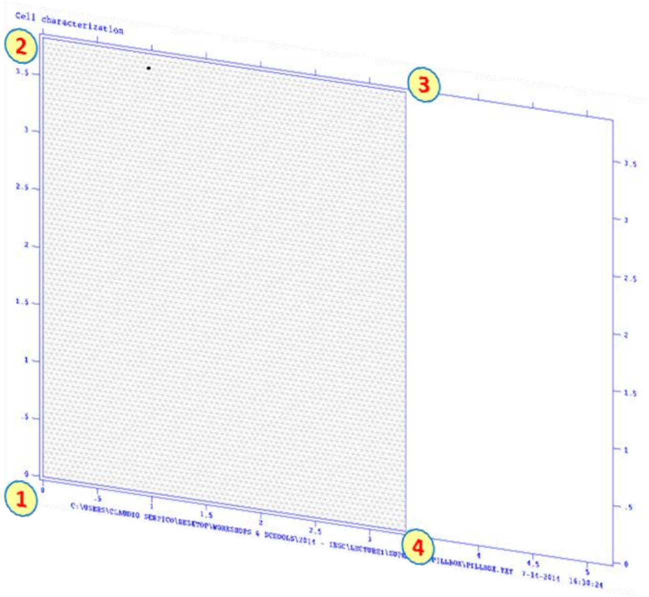

Pillbox.TXT - Blocco note
File Modifica Formato Visualizza ?

```
cell characterization

$reg kprob= 1 ,                          ; Superfish problem
dx= 5.000000e-02 ,                       ; x mesh spacing
freq= 3000 ,                             ; starting frequency in MHz

xdri= 1.00000e+00 ,                      ; drive point x-location
ydri= 3.7e+00 $                          ; drive point x-location

$po x= 0 , y= 0 $                                  ; Start of bounday points
$po x= 0e+00 , y= 3.827e+00 $
$po x= 3.333e+00 , y= 3.827e+00 $
$po x= 3.333e+00 , y= 0e+00 $
$po x= 0 , y= 0 $
```



A Pillbox cavity with Superfish

Pillbox.TXT - Blocco note

```
Cell characterization

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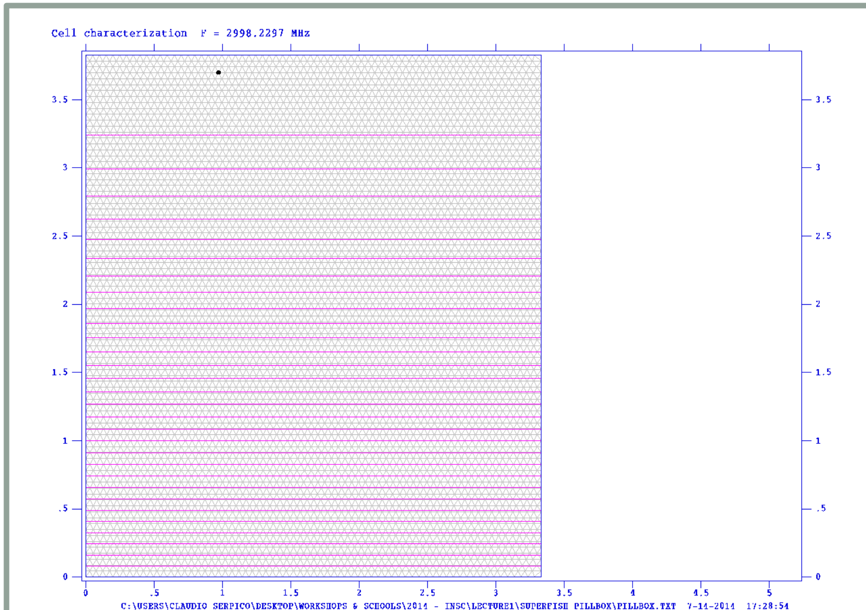
$po x= 0 , y= 0 $
```



PILLBOX.SFO - Blocco note

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Superfish output summary for problem description:
Cell characterization
Problem file: C:\USERS\CLAUDIO SERPICO\DESKTOP\WORKSHOPS & SCHOOLS\2014 - INSC\LECTURE1\SUPERFISH
PILLBOX\PILLBOX.TXT  7-14-2014  17:28:54
--------------------------------------------------------------------------------
All calculated values below refer to the mesh geometry only.
Field normalization (NORM = 0):      EZERO =      1.00000 MV/m
Frequency                                  =   2998.22969 MHz
Particle rest mass energy                  =   938.272029 MeV
Beta =  0.6666678           Kinetic energy =      320.554 MeV
Normalization factor for E0 =  1.000 MV/m =     5354.223
Transit-time factor                        =    0.0000017
Stored energy                              =  1.82785E-04 Joules
Using standard room-temperature copper.
Surface resistance                         =     14.28544 milliOhm
Normal-conductor resistivity               =      1.72410 microOhm-cm
Operating temperature                      =      20.0000 C
Power dissipation                          =     108.6013 W
Q     =    31706.6          Shunt impedance =      306.902 MOhm/m
Rs*Q  =    452.943 Ohm               Z*T*T =        0.000 MOhm/m
r/Q   =      0.000 Ohm  Wake loss parameter =      0.00000 V/pC
Average magnetic field on the outer wall   =      1377.37 A/m, 1.35508 W/cm^2
Maximum H (at Z,R = 3.15889,3.827)         =      1377.37 A/m, 1.35508 W/cm^2
Maximum E (at Z,R = 3.30813,3.827)         =  7.35819E-05 MV/m, 1.5729E-06 Kilp.
Ratio of peak fields Bmax/Emax             =   23522.8467 mT/(MV/m)
Peak-to-average ratio Emax/E0              =       0.0001

Wall segments:
Segment    Zend        Rend         Emax      Power      P/A        dF/dz     dF/dR
           (cm)        (cm)        (MV/m)      (W)     (W/cm^2)   (MHz/mm)  (MHz/mm)
-----------------------------------------------------------------------------------------
          0.0000      3.8270
   2      3.3330      3.8270    9.0256E-05   108.6      1.355      0.000     -78.35
-----------------------------------------------------------------------------------------
                                   Total     108.6
```

References

1. ‘Electromagnetic Wave and Antennas’, S. J. Orfanidis, Rutgers University
2. ‘Cavity Basics’, E. Jensen, Cern Accelerator School, CERN-2011-007
3. ‘RF Linear Accelerators’, T.P. Wangler, Wiley-VCH
4. ‘RF Cavity Design with Superfish’, C. Plostinar, John Adams Institute