ELECTRON LINAC:

DESIGN AND PERFORMANCE OF ACCELERATING STRUCTURES

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Outline

- The Lorentz Force Equation
- Group Velocity and Phase Velocity
- Energy Velocity
- RF Parameters for a Resonant Cavity
- Periodic Accelerating Structures
- Traveling-wave Linac Structures
- Empirical Design with Superfish
- FERMI BTW Accelerating Structures
- Longitudinal tracking of e-beam

The Lorentz Force Equation

For a particle of charge q and velocity \vec{v} in an electric field \vec{E} and a magnetic field \vec{B} , the Lorentz force is given by:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

• $\vec{F}_E = q\vec{E}$ is the electric field force: this force has the same direction of the electric field.

The electric field can be used to vary the energy of a charged particle beam.

• $\vec{F}_B = q\vec{v} \times \vec{B}$ is the magnetic field force: this force is always perpendicular to the particles' trajectory.

The magnetic field can be used to bend the trajectory of a charged particle beam.

Group Velocity and Phase Velocity^[1]

There are no truly monochromatic waves in nature. A real wave exists in the form of a wave group, which consists of a superposition of waves of different frequencies and wave numbers. If the spread in the phase velocities of the individual waves is small, the envelope of the wave pattern will tend to mantain its shape as it will moves with a velocity that is called *group velocity*.

The simplest case of a wave group, consists of two equal-amplitude waves, propagating in the +z direction, with frequencies ω_1 and ω_2 , and wave numebers k_1 and k_2 :

$$V(z,t) = e^{j(\omega_1 t - k_1 z)} + e^{j(\omega_2 t - k_2 z)} = 2\cos\left[\frac{(\omega_1 - \omega_2)t - (k_1 - k_2)z}{2}\right]e^{j[(\omega_1 + \omega_2)t - (k_1 + k_2)z]/2}$$

The exponential factor describes a traveling wave with the mean frequency and mean wave number,.

The first factor represents a slowly varying modulation of the wave amplitude.

Group Velocity and Phase Velocity

The *phase velocities* of component wave are ω_1/k_1 and ω_2/k_2 , and the mean *phase velocity* is:

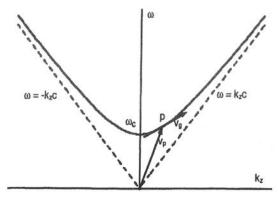
$$v_p = \frac{\omega_1 + \omega_2}{k_1 + k_2} = \frac{\overline{\omega}}{\overline{k}}$$

The *group velocity* is defined as the velocity of the amplitude-modulation envelope: $\omega_1-\omega_2 \quad \partial \omega$

 $v_g = \frac{\omega_1 - \omega_2}{k_1 - k_2} \to \frac{\partial \omega}{\partial k}$

The *phase velocity* at any point on the curve is the slope of the line from the origin to that point. The *group velocity* is the slope of the dispersion curve at that point.

For the uniform waveguide $v_p v_g = c^2$, where $v_g < c$ and $v_p > c$.



Dispersion curve for uniform waveguide, $\omega^2 = \omega_c^2 + (k_z c)^2$

Energy Velocity

The *energy velocity* is defined as the velocity of electromagnetic energy flow. For a traveling wave moving in the +z direction the *energy velocity* is given by:

$$v_E = \frac{P_+}{U_l}$$

where P_+ is the wave power, the electromagnetic energy per unit time crossing a transverse plane at fixed z, and U_l is the stored electromagnetic energy per unit length.

Suggestion:

In practical cases, the *energy velocity* is equal to the *group velocity*. The previous forumla can be used for post-processing the results of high frequency simulations (i.e. HFSS, Microwave Studio) to evaluate the *group velocity* (i.e. the *filling time*) of accelerating cavities.

RF Parameters for a Resonant Cavity Stored Energy and Quality Factor

The total energy stored in a cavity is the sum of the electric field energy and the magnetic field energy.

$$W = \iiint_{cavity} \frac{\varepsilon}{2} |\vec{E}|^2 dV + \iiint_{cavity} \frac{\mu}{2} |\vec{H}|^2 dV$$

Electric field energy Magnetic field energy

The quality factor Q is defined as:

$$Q = \frac{\omega_0 W}{P_{loss}}$$

Here ω_0 denotes the eigenfrequency and W the stored energy. P_{loss} is the power lost into the cavity walls (or any other loss mechanism). It is clear that the larger the Q, the smaller will become the power necessary to compensate for cavity losses.

RF Parameters for a Resonant Cavity Accelerating Voltage and Transit Time factor

The accelerating voltage is defined as:

$$V_{acc} = \int_{-\infty}^{\infty} \vec{E}(z) e^{j\frac{\omega}{\beta c}z} dz$$

The exponential accounts for the movement of the particle with speed βc through the cavity while the fields continue to oscillate.

In the definition of the accelerating voltage we already accounted for the finite speed of the particles through the cavity. It thus includes already the so-called transit-time factor, which describes this effect alone. The transit-time factor T is defined as:

$$T = \frac{|V_{acc}|}{\int_{-\infty}^{\infty} |\vec{E}(z)| dz}$$

RF Parameters for a Resonant Cavity Shunt Impedance & R/Q

The *shunt impedance* is a figure of merit that measures the effectiveness of producing an axial voltage for a given power dissipation.

$$R = \frac{|V_{acc}|^2}{P_d L} = \frac{|V_0 T|^2}{P_d L} \qquad \left[\frac{M\Omega}{m}\right]$$

(effective shunt impedance per unit length)

The quantity *R*-upon-Q is defined as:

$$\frac{R}{Q} = \frac{|V_{acc}|^2}{\omega_0 W}$$

The R/Q quantifies how effectively the cavity converts stored energy into acceleration. The R/Q is uniquely determined by the geometry of the cavity.

In a lossless uniform waveguide with azimuthal symmetry, the axial electric field for the lowest transverse-magnetic mode, the TM₀₁ mode is:

$$E_z(r, z, t) = EJ_0(Kr)e^{j(\omega t - k_0 z)}$$

The previous formula describes a wave propagating in the +z direction, with wavenumber $k_0 = 2\pi/\lambda_g$ where λ_g is the guide wavelength. The uniform waveguide dispersion relation is given by:

$$\omega^2 = \omega_c^2 + (k_0 c)^2 = (Kc)^2 + (k_0 c)^2$$

where K is the cutoff wavenumber for the TM_{01} mode. The *phase velocity* can be then expressed as

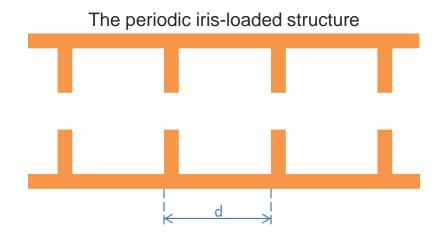
$$v_p = \frac{\omega}{k_0} = \frac{c}{\sqrt{1 - (Kc)^2/\omega^2}} > c$$

Since the *phase velocity* is always larger than *c*, a uniform waveguide cannot be used for synchronous-particle acceleration.

Periodic Accelerating Structures Floquet Theorem and Space Harmonic

"In a given mode of an infinite periodic structure, the fields at two different cross sections that are separated by one period differ only by a constant factor, which in general is a complex number":

- There are intervals of ω where the constant is real with magnitude less than 1. For these regions the modes are evanescent (i.e. stopband).
- There are also *passbands* in which waves will propagate. In the *loss-free* case the complex constant is e^{jk_0d} , which physically represent a cell-to cell phase shift k_0d of the field.



Periodic Accelerating Structures Floquet Theorem and Space Harmonic

The Floquet theorem in a passband is expressed as:

$$E(r, z + d) = E(r, z)e^{\pm jk_0d}$$

Since E(r,z) is periodic, it can be expressed in a Fourier series. So, the solution for the propagating wave can be expressed as:

$$E(r,z,t) = \sum_{n=-\infty}^{\infty} E_n J_0 \left[\left(\left(\frac{\omega}{c} \right)^2 - k_n^2 \right) r \right] e^{j(\omega t - k_n z)}$$

where $k_n = k_0 + 2\pi n/d$.

The previous equation represents the sum of an infinite number of traveling waves (i.e. *space harmonics*) denoted by the index *n*.

The space harmonics have the same frequency but different wavenumbers.

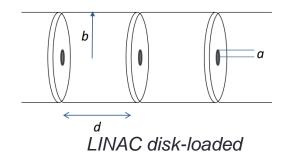
Periodic Accelerating Structures Floquet Theorem and Space Harmonic

The phase velocity for the nth space harmonic is:

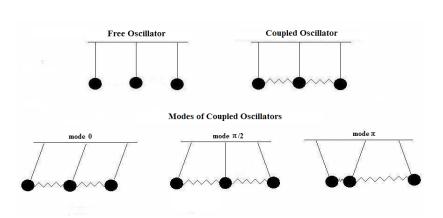
$$v_{p,n} = \frac{\omega}{k_n} = \frac{v_{p,0}}{1 + (2\pi n/k_0 d)}$$

- By choosing n sufficiently large, one can obtain an arbitrarly low phase velocity.
- The introduction of periodicity led not only to the generation of *space harmonics*, but also modified the *phase velocity* of the foundamental wave $v_{p,0} = \omega/k_0$. Physically, this can be attributed to the addition of reflections from the period elements to the principle wave.

A single cavity such as a *pillbox cavity* has an infinite number of *resonant modes* (i.e. *cavity modes*) named *transverse-electric* (TE_{mnp}) and *transverse-magnetic* (TM_{mnp}) modes.



For any system of coupled oscillators, there exists a family of so called *normal-modes*, each mode behaving like an independent harmonic oscillator with its own characteristic resonant frequency.



In general, when any normal mode is excited by a suitable driving force at the right frequency, each of the individual oscillators participates in the motion, and each oscillates at the same frequency with a characteristic phase difference from one oscillator to the next.

Traveling-wave Linac Structures

Selection of operating mode^[2]

The efficiency of the structure as an accelerator of electrons is measured by the shunt impedance per unit length.

An approximate equation for the shunt impedance per unit length is:

$$R = 968 \left(\frac{\beta_w}{\delta}\right) \frac{(1-\eta)^2}{n+2.61\beta_w(1-\eta)} \left(\frac{\sin \frac{D}{2}}{\frac{D}{2}}\right)^2$$

where:

- β_w is the *phase velocity* in the structure divided by c
- δ is the *skin depth*
- t is the disk thickness
- d is the periodo of the structure
- η is equal to t/d
- n is equal to λ/d , the number of disks per wavelength
- *D* is equal to $(2\pi/\lambda)(d-t)$

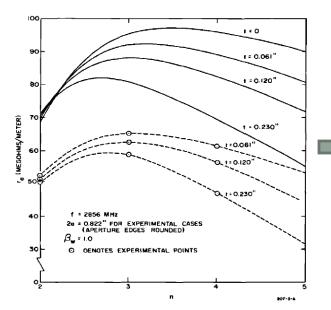
Traveling-wave Linac Structures

Selection of operating mode

The previous equation gives too high value of *R* for two reasons:

- The conductivity of the copper walls is never as high as the idealized value used in calculating the numerical constant.
- The effect of the disk apertures is not taken into account.

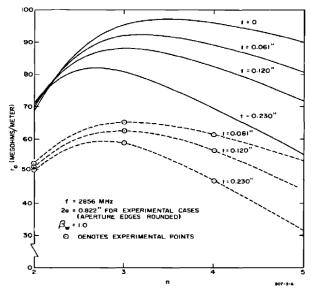
Neverthless the relative variation of *R* with the spacing and the thickness of the disks was confirmed by experimental measurements.



- a. The *shunt impedance* of the individual cavities is improved by increasing the disk spacing.
- b. The fraction of the length available for accelerating fields to act on the electrons is increased as the number of disks per wavelength is decreased.
- c. As the disk spacing is decreased, the electron transit time is decreased correspondingly, and the 'effective' field strength acting on the electron is increased.

Traveling-wave Linac Structures

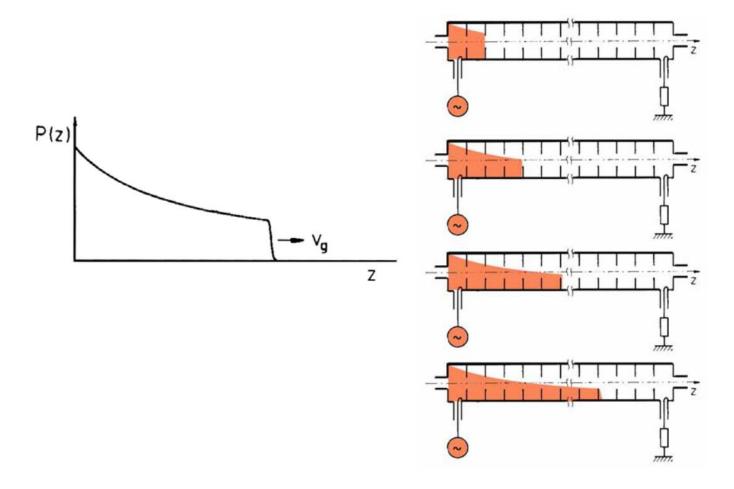
Selection of operating mode



- For negligible disk thickness $(t \approx 0)$, the optimum number of disks per wavelength is approximately 3.5.
- For a disk thickness of about 3 mm (i.e. 0.12 inch) the best value is about n = 3.

The value of n=3 is typically adopted in travelling wave accelerating structures and corresponds to a phase shift of $2\pi/3$ radians per cavity.

Energy flow in traveling wave linac^[3]



Energy flow in traveling wave linac

$$v_{energy} = v_g$$

From the definition of Q

$$\frac{dP_t}{dz} = -\frac{\omega P_t}{v_a Q} = -2\alpha P_t \quad ; \quad \frac{dE_a}{dz} = -\alpha E_a$$

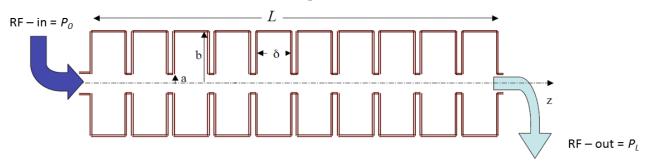
where the attenuation length is defined as:

$$\alpha = \frac{\omega}{2v_g Q}$$

then:

$$E_a^2 = R_{sh} \left| \frac{dP_t}{dz} \right| = 2\alpha R_{sh} P_t$$

Constant-impedance^[3]



A structure with constant structure parameters along its length

In such a case we have:

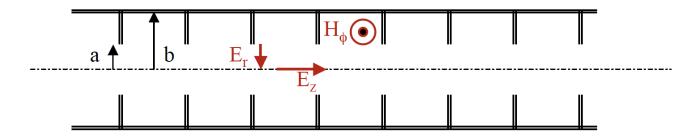
$$E_a(z) = E_0 e^{-\alpha z}$$
 ; $P_t(z) = P_0 e^{-2\alpha z}$

For a structure of length *L* the attenuation parameter is:

$$\tau = \alpha L = \frac{\omega L}{2v_g Q}$$

Transmitted power and accelerating gradient decrease exponentially along the structure.

Constant-gradient



In the region of the aperture:

$$E_r \propto r$$
 ; $H_{\Phi} \propto r$

The power flowing through the structure is:

$$P_t = \int_0^a (E \times H) r dr \propto a^4 \implies v_g \propto a^4$$

Small variations in a lead to large variations in v_g and P_t

Constant-gradient

$$E_a^2 = R_{sh} \left| \frac{dP_t}{dz} \right| = 2\alpha R_{sh} P_t$$

It is possible to make E_a constant by varying P_t as α^{-1} .

Since R_{sh} varies weakly with the iris size, then:

$$\left| \frac{dP_t}{dz} \right| = const \implies P_t(z) = P_0 - (P_0 - P_L)(z/L)$$

We get:

$$\frac{P_t(z)}{P_0} = 1 - (z/L)(1 - e^{-2\tau})$$

Constant-gradient

$$\left| \frac{dP_t}{dz} \right| = -\frac{P_0}{L} (1 - e^{-2\tau})$$

Considering that:

$$\frac{dP_t}{dz} = -\frac{\omega P_t}{v_a Q} = -2\alpha P_t$$

So:

$$v_g = -\frac{\omega P_t}{Q \frac{dP_t}{dz}} = \frac{\omega L P_t}{Q P_0 (1 - e^{-2\tau})}$$

$$v_g = rac{\omega L}{Q} rac{\left(1 - rac{Z}{L} \left(1 - e^{-2 au}
ight)
ight)}{\left(1 - e^{-2 au}
ight)}$$

To get a constant gradient along the accelerating structure is necessary to reduce the *group velocity* along $z \implies$ make the irises smaller

CI vs CG structures^[2]

Constant-impedance structures

- Uniform dimension of the accelerating cells.
- Exponential decay of the power and field strength with axial distance form the input end.
- The ratio of maximum-to-average axial electric field is given by:

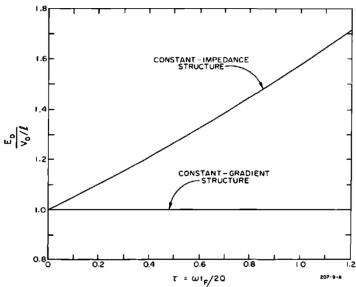
$$\frac{E_0}{V_0/l} = \frac{\tau}{1 - e^{-\tau}}$$

where τ is the RF attenuation parameter equal to $\tau = \omega t_f/2Q$.

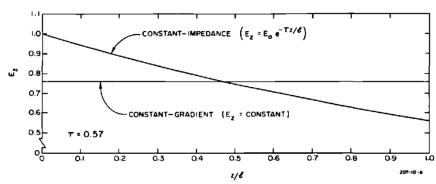
Constant-gradient structures

- Non-Uniform dimension of the accelerating cells
- Axial fields remain constant over the enitre length.
- The ratio of maximum-to-average axial electric field is equal to 1.

CI vs CG structures



Ratios of maximum-to-average axial electric field strengths in constant-impedance and constant-gradient accelerator structures verus τ



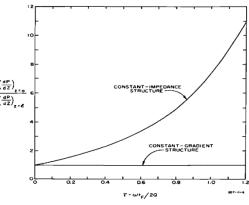
Axial field strength versus z/l for equal electron energy gain in constant-gradient and constant-impedance sections

The constant-gradient structure can produce higher electron energies than an optimized constant-impedance structure when both are operating at the breakdown limit of electro-magnetic field strength.

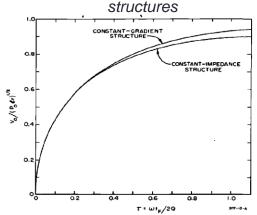
CI vs CG structures

Advantages over the constant-impedance structure:

- The power dissipated per unit length in the constant-gradient accelerator is constant over the entire length of the structure. In contrast, the ratio of power loss at the input end to that at the output end of a constant-impedance structure may be as high as 12 to 1.
- The constant-gradient structure gives a slightly higher no-load beam energy than the constant-impedance structure. Also, the constant-gradient structure has greater relative energy advantage in the loaded case than in the unloaded case.



Ratio of power losses at input and output ends versus τ for CI and CG

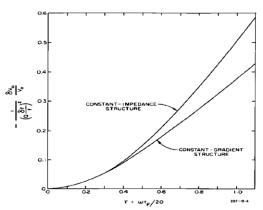


Unloaded beam energy versus τ for CI and CG structures

CI vs CG structures

Advantages over the constant-impedance structure:

- The constant-gradient structure has a higher maximum conversion efficiency (ratio of maximum electron beam power to input RF power) than the constant-impedance structure.
- The constant-gradient accelerator is less frequency-sensitive than the constant-impedance accelerator.



Ratio of power losses at input and output ends versus τ for CI and CG structures

Disadvantages over the constant-impedance structure:

 The nonuniform modular dimensions made the cavities in the constantgradient structure more expensive to fabricate and to test.

Empirical Design

The disk-loaded waveguide is not the only slow-wave structure capable of accelerating electrons. In fact, other structures may yeld shunt impedance about twice that of the disk-loaded waveguide.

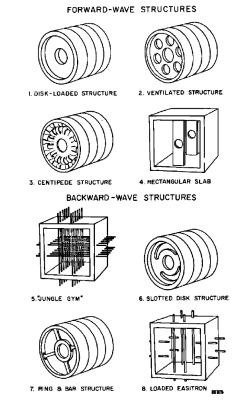
But where a large improvement in the shunt impedance can be obtained, the resulting v_g can be 10 times as high as desired.

Pros

High Shunt Impedance

Cons

- High group velocity
- Recirculation or extreme lengths between feeds required



Disk-loaded waveguide are generally preferred

Empirical Design

- For the disk-loaded waveguide an improvement of the Shunt Impedance can be gained by adopting the $2\pi/3$ mode.
- For standard accelerating sections (i.e. we refer to electron linac), the target is to obtain a *phase velocity* equal to the velocity of light c. Thus, specifying the operating mode and the frequency (or equivalently the free-space *wavelength*) the distance between the center lines *d* is fixed as follow:

$$v_p = \frac{\omega}{k_0} = c$$

considering that

$$k_0 d = \phi_0$$

So we get:

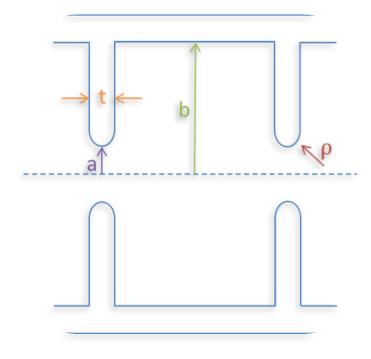
$$d=\phi_0$$
 ϕ_0 is the phase advance per

$$d = \frac{\phi_0 c}{\omega_0}$$

Empirical Design with Superfish

Once ϕ_0 (i.e. the *phase advance per cell*) and thus the cell length d is fixed, 4 dimensions remain to be specified:

- 2b, the outer cell radius
- 2a, the beam iris aperture
- *t*, the disk thickness
- ρ , the radius of curvature

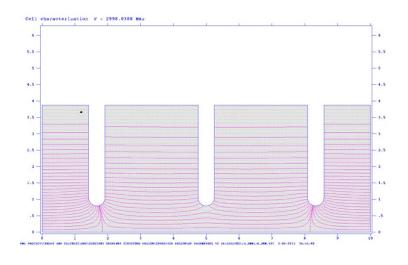


The optimal geometric parameters can be at first evaluated using Supefish.

Empirical Design with Superfish

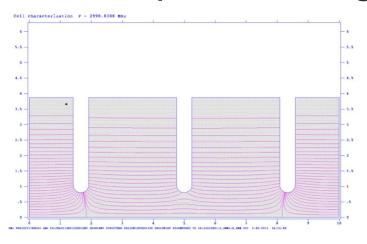
- Simulating more than 1 cell, we will be able to get the different normalmodes, each one characterized by its own phase advance (see Slide n.14).
- In case of half-cell termination (i.e. the simulated geometry is closed with half-cells and a perfect electric conductor boundary condition), the phase advance per cell is given by:

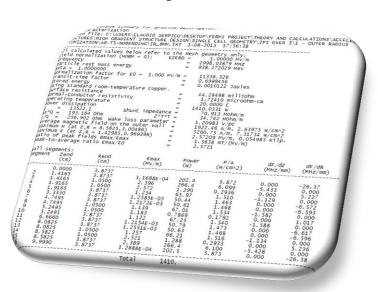
$$\varphi_m = \frac{(m-1)\pi}{N-1} \quad \text{with} \quad \mathbf{m} = 1 \dots N$$



Using Superfish, to get the $2\pi/3$ mode we shall simulate a geometry made by 4 accelerating cells.

Empirical Design with Superfish

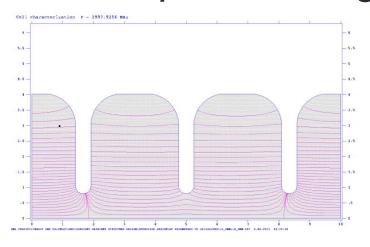




Iris Radius: 8mm Disk Thickness: 5 mm No Bending Radius

Parameter	Simulated Value	Units	
$egin{aligned} \mathbf{f_0} \ \mathbf{Q_0} \ \mathbf{R_{sh}} \end{aligned}$	2998.01	MHz	
\mathbf{Q}_0	13522		
R_{sh}	70.91	$M\Omega/m$	
v_g/c R_{sh}/Q_0	0.42	%	
R_{sh}/Q_0	5244		
E_0	50	MV/m	
E _{peak} (axis) E _{max} (iris)	95.3	MV/m	
E _{max} (iris)	128.5	MV/m	
${ m E_{max}/E_{peak}} \ { m E_{max}/E_0}$	1.34		
E_{max}/E_0	2.57		

Empirical Design with Superfish



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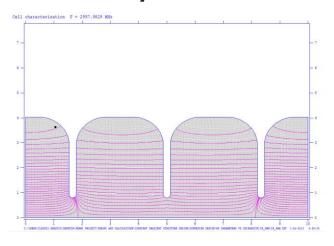
Outer bending radius to reduce power losses:

- Q factor increased
- R_{sh} increased as well

Iris Radius: 8 mm Disk Thickness: 5 mm Bending Radius: 11 mm

Parameter	Simulated Value	Units	
$\mathbf{f}_{_{0}}$	2998.01	MHz	
Q_o	14962		
\mathbf{R}_{sh}	77.99	$M\Omega/m$	
v _g /c	0.42	%	
R_{sh}/Q_0	5212		
\mathbf{E}_{0}	50	MV/m	
E _{peak} (axis)	95.3	MV/m	
E _{peak} (axis) E _{max} (iris)	125.5	MV/m	
$E_{\text{max}}/E_{\text{peak}}$	1.31		
$rac{E_{ ext{max}}/E_{ ext{peak}}}{E_{ ext{max}}/E_{ ext{0}}}$	2.51		

Empirical Design with Superfish





Disk thickness t reduced:

- R_{sh} increased
- v_a increased
- Mechanical strength should be verified

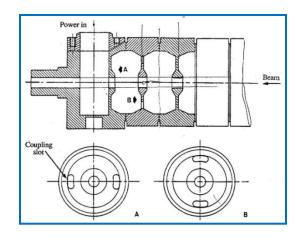
Iris Radius: 8mm Disk Thickness: 2.5 mm Bending Radius: 11 mm

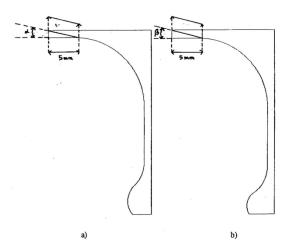
Parameter	Simulated Value	Units	
\mathbf{f}_0	2998.01	MHz	
Q_0	15730		
$\begin{array}{c} Q_0 \\ R_{sh} \\ v_g/c \\ R_{sh}/Q_0 \end{array}$	83.6	$\mathrm{M}\Omega/\mathrm{m}$	
v_{g}/c	0.708	%	
$R_{\rm sh}/Q_0$			
\mathbf{E}_{0}	50	MV/m	
E _{peak} (axis) E _{max} (iris)	92.22	MV/m	
E _{max} (iris)	136	MV/m	
$\frac{E_{max}/E_{peak}}{E_{max}/E_0}$	1.47		
E /E	2.72		

THE FERMI LINAC BTW ACCELERATING STRUCTURES

BTW accelerating structures

S-type structures are Backward Traveling Wave (BTW) structures comprised of 162 nose cone cavities coupled magnetically.





Туре	Const. gradient, mag. coupled	
Mode	$3\pi/4$	
Frequency	2998.01	MHz
Length	6.15	m
Q	11000	
$R_{\rm sh}$	71-73	MOhm/m
Filling time	0.747	μ s

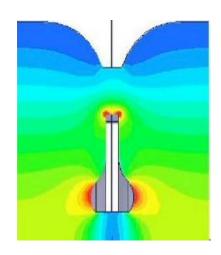
BTW accelerating structures Electrical Behaviour

Past operation of the BTW accelerating structures showed that such structures suffered heavy breakdown phenomena when pushed to high gradient (no phase modulation implemented).



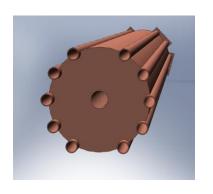


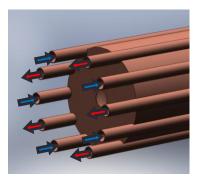
Single cell simulation shows that surface electric field on the magnetic coupling slot becomes of the same magnitude and even higher than the surface electric peak field on the nose.



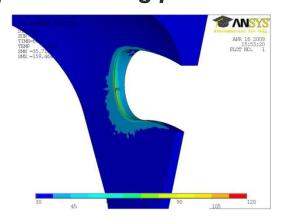
BTW accelerating structures

Thermo-mechanical Behaviour

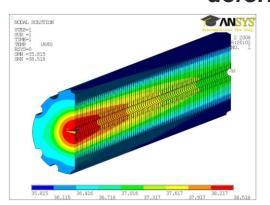


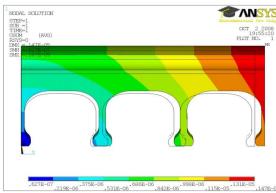


RF pulse-heating phenomena



Steady-state thermal profile and mechanical deformation

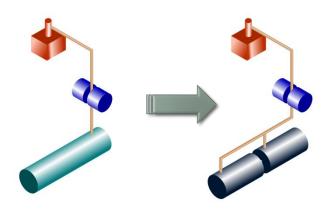




New accelerating structures

RF plant configuration

The module will be comprised of two 3-m long accelerating structures



Acc. module length	6	m
Kly. pulse duration	4000	ns
Q_0	190000	
Q _{ext}	19000	

Optim	al R	F pa	aramete	ers
have	been	eva	luated	to
benef	it	from	pu	lse
compi	ressio	n.		



"Modified Poynting vector" on cell's surface has been evaluated for a 25 MV/m accelerating gradient.



а	10	mm
Q_0	14900	
R_{sh}	71.7	MΩ/m
Filling Time	672	ns
v _g /c	1.49	%

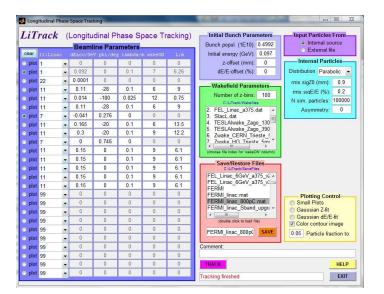


 $Sc_max = 0.7 W/\mu m^2$

linac it will be 8e-14 bpp.

LiTrack^[4]

- LiTrack is a Matlab code with additional features, such as graphical user interface, prompt output plotting, and functional call within a script.
- It is a macro-particle tracking program, that follows longitudinal position z (bunch head at z < 0) and relative energy deviation $\delta = \Delta E/E_0$ (where E_0 is reference energy) of the particles. It can be used to track ultra-relativistic beams, where space charge can be ignored, through linacs and transport lines, where initial conditions and the beamline elements determine the final phase space.



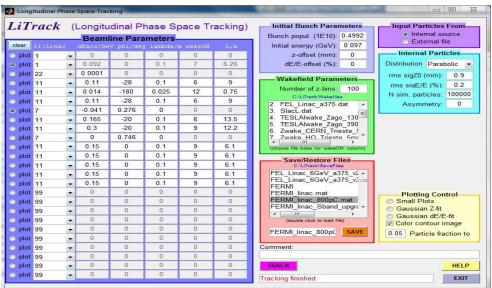
List of function-codes and their parameters

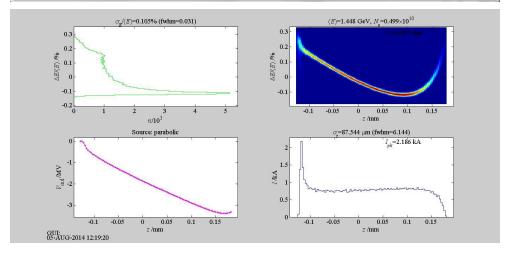
descrip.	code	p_1	p_2	p_3	p_4	p_5
mark	1					
dump	2					
compr.	6	R_{56}	T_{566}	E_0	U_{5666}	
chicane	7	R_{56}	E_{0}			
linac	10	E_{final}	ϕ	λ	i_{wake}	L
linac	11	V_0	ϕ	λ	i_{wake}	L
rw-wake	15	r	L	σ_c	au	1/0
ISR	22	σ_{δ}				
E-cuts	26	δ_{min}	δ_{max}			
z-cuts	36	z_{min}	z_{max}			
STOP	99					

BTW accelerating structures

Longitudinal wakefields effect

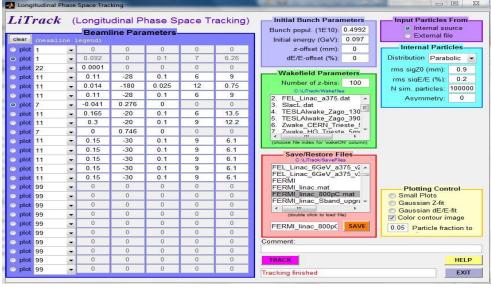
Bunch & Linac Parameters				
Bunch Charge	800	pC		
rms $\sigma_{\!z_0}$	0.9	mm		
rms σ_E/E	0.2	%		
BC1 compression factor	~10			
Linac3 & Linac4 Acc. Gradient	25	MV/m		

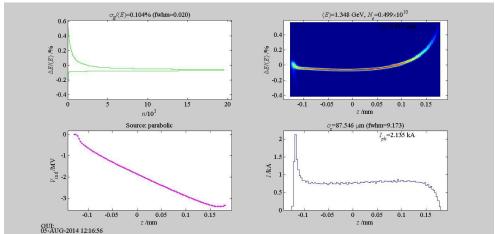




BTW accelerating structures

Longitudinal wakefields effect







To get a Bunch Current of about 800 A, with an Energy Spread of 0.2 %, it is necessary to put off-crest some accelerating structures in the last part of the Linac

The energy loss could be up to 130 MeV.

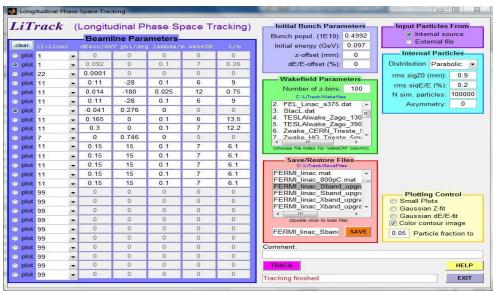
New accelerating structures

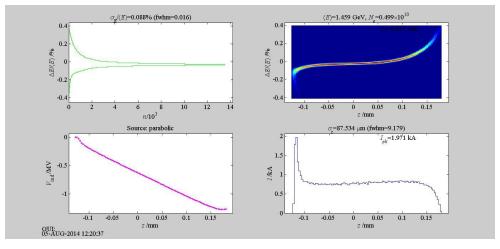
Longitudinal wakefields effect

Bunch & Linac Parameters				
Bunch Charge	800	pC		
rms $\sigma_{\!z_0}$	0.9	mm		
rms σ_E/E	0.2	%		
BC1 compression factor	~10			
Linac3 & Linac4 Acc. Gradient	25	MV/m		



We can get the target e-beam energy with the required "flat" phase space





References

- 1. 'RF Linear Accelerators', T.P. Wangler, Wiley-VCH.
- 2. 'The Stanford Two-Mile Accelerator', W.A. Benjamin Inc., 1968.
- 3. 'Acceleration by RF waves', W. A. Barletta, United States Particle Accelerator School.
- 4. 'LiTrack: a fast longitudinal phase space tracking code with graphical user interface', K. Bane & P. Emma, 2005