Controlled thermonuclear with tokamak

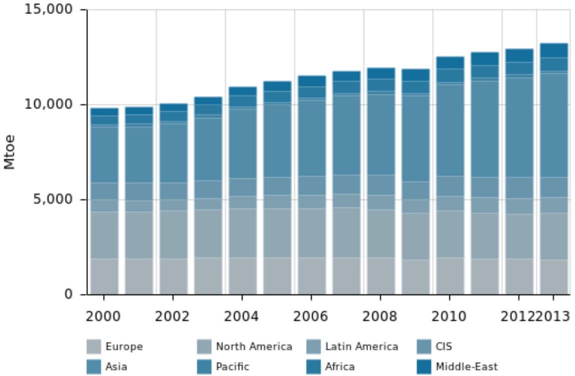
Richard Marchand
University of Alberta, Canada
Richard.Marchand@ualberta.ca

International Nathagali Summer College
August 4-16 2014

Outline

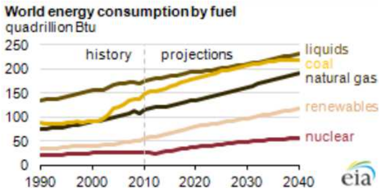
1. Introduction: Problems with existing energy sources
2. Alternative: Nuclear fusion
3. Basic plasma physics
4. Outlook

World energy consumption



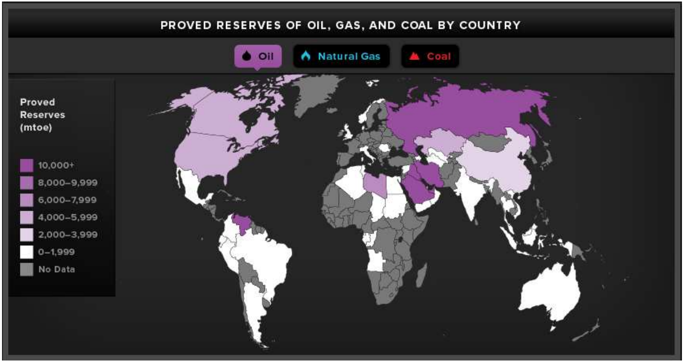
Mtoe = “million tonne of oil equivalent”
(http://yearbook.enerdata.net/)

Energy consumption by type



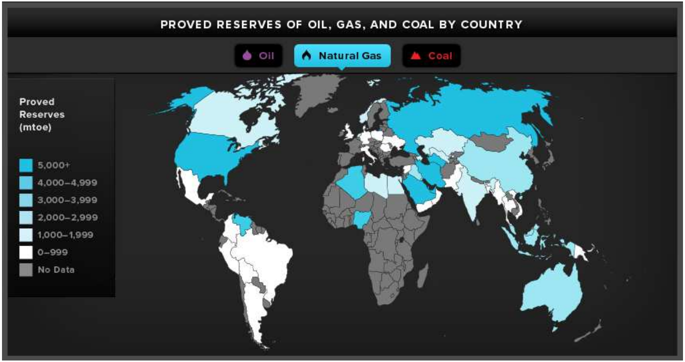
(http://www.eia.gov)

Fossil fuel reserves - Oil

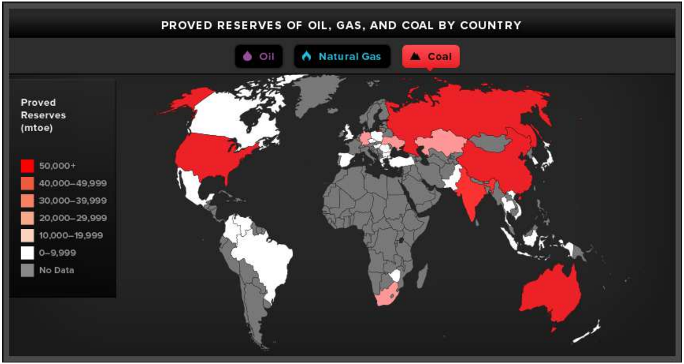


http://www.energyrealities.org/chapter/our-resources/item/proved-reserves-of-fossil-fuels/erp6F0E6DFD5D4365155

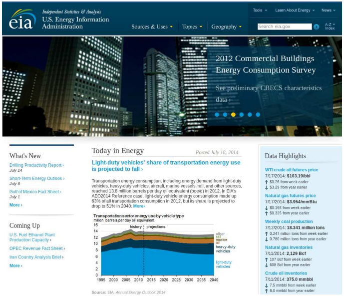
Fossil fuel reserves - Gas



Fossil fuel reserves - Coal



Numbers



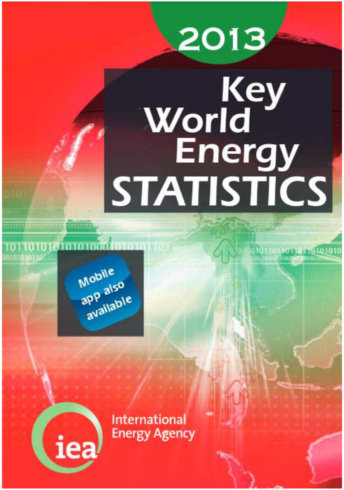
http://www.eia.gov/

EEA Technical report | No 17/2013

Climate and energy country profiles —
Key facts and figures for EEA member countries

ISSN 1725-2237

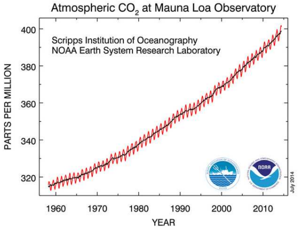
http://www.eea.europa.eu/



http://www.iea.org/

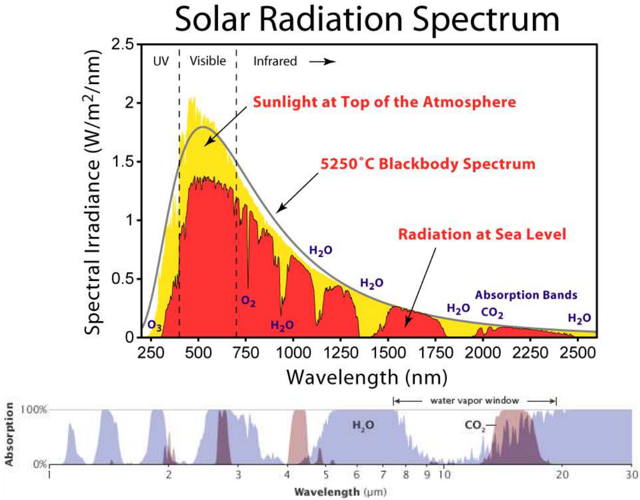
Arithmetic

- Total energy reserves $\sim 2000\mathrm{Mtoe}$
- Assume a steady consumption of $\sim 15\mathrm{Mtoe}$ per year
- $\rightarrow \gtrsim 130$ years worth of supplies.
- Problem: Green house gases and climate changes.



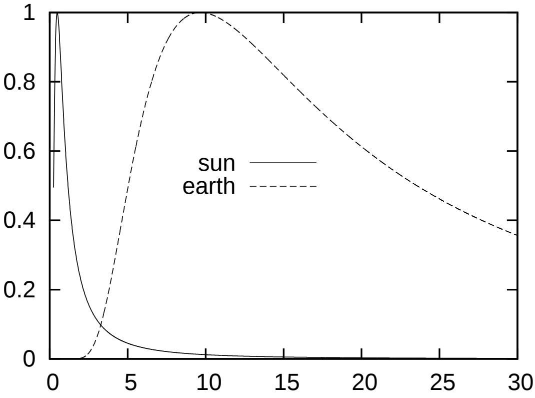
(wikipedia)

Green house effect



(wikipedia)

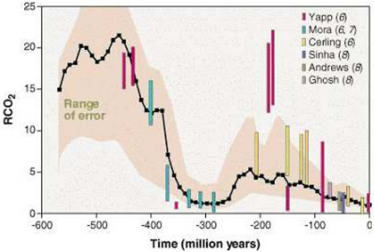
Incident and radiated spectrum



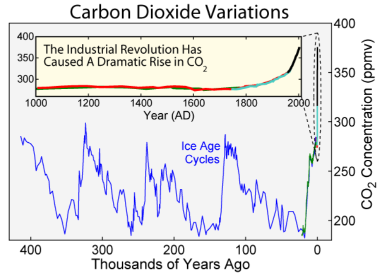
Normalized black body radiation at $5800\mathrm{K}$ (sun) and $16^{\circ}\mathrm{C}$ (Earth).

The radiation spectrum from Earth is shifted toward longer wavelengths, compared to that of the sun.

Is CO_2 really bad?



http://earthguide.ucsd.edu/virtualmuseum/climatechange2/07_1.s

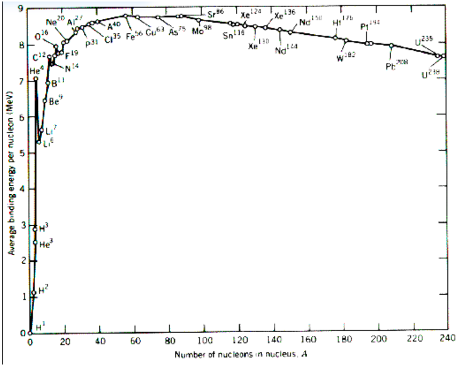


(wikipedia)

What are the alternatives?

- Do nothing
- Nuclear
- Renewable
  - Hydro
  - Wind
  - Tides
  - Solar
  - Geothermal

Nuclear energy



Exothermic nuclear reactions:

- $A < 26$: Fusion
- $A > 26$: Fission

Fission

☺

- Relatively easy to achieve.
- Much more efficient than fossil fuels.
- Can generate high power densities.
- Can be close to cities and industrial centres.

☹

- Activates radioactive materials ($\gtrsim 100$ years).
- Limited supplies: $\gtrsim 500$ years; more with breeder reactors.
- Produces long term toxic and radioactive waste ($> 10^4$ years).
  See the movie "Into Eternity".

Fusion

☺

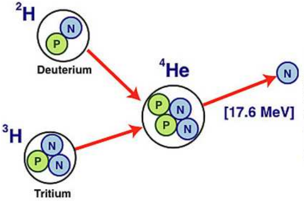
- Energetically much more efficient.
- Fuel is abundant and practically limitless.
- Produces short lived ($\lesssim$ 100 years) radioactive materials.

☹

- Technically much more difficult.

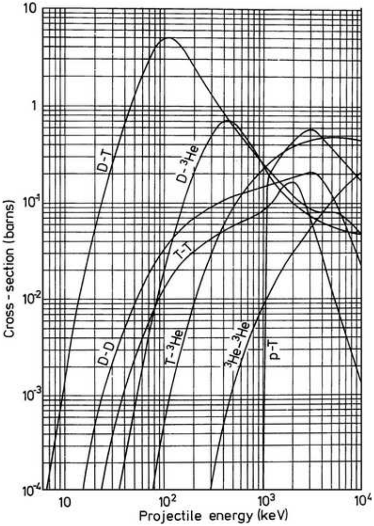
How does it work?

- D-T reaction



- Aneutronic reactions

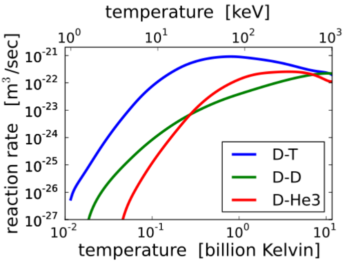
^{2}D	+ ^{3}He →		^{4}He	+	^{1}p	+ 18.3 MeV
^{2}D	+ ^{6}Li	→ 2	^{4}He			+ 22.4 MeV
^{1}p	+ ^{6}Li	→	^{4}He	+	^{3}He	+ 4.0 MeV
^{3}He	+ ^{6}Li	→ 2	^{4}He	+	^{1}p	+ 16.9 MeV
^{3}He	+ ^{3}He →		^{4}He	+ 2	^{1}p	+ 12.86 MeV
^{1}p	+ ^{7}Li	→ 2	^{4}He			+ 17.2 MeV
^{1}p	+ ^{11}B	→ 3	^{4}He			+ 8.7 MeV
^{1}p	+ ^{15}N	→	^{12}C	+	^{4}He	+ 5.0 MeV[2]



(Wikipedia)

Lawson's condition

- Consider D-T for which the cross section is the highest.
- The maximum cross section is obtained at energy $E \simeq 10^5 eV$



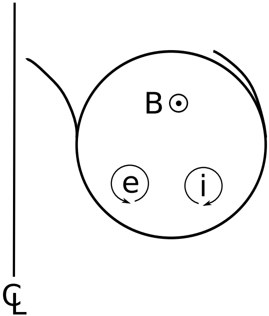
(wikipedia)

- From $E = \frac{e^2}{4\pi\epsilon_0} d$, in a 'head on' collision, the two nuclei come $\sim 14 fm$ apart.
- Reaction rates for a thermal plasma involves an average over a Maxwellian velocity distribution.
- In order to 'ignite' at $T \sim 20\mathrm{keV}$ a fusion plasma must satisfy $n\tau_E \gtrsim 3 \times 10^{19}\mathrm{sm}^{-3}$.

Possible approaches

- Cold fusion
- Muon-catalyzed fusion
- Magnetic confinement
  - Toroidal devices: stellerators, tokamaks, bumpy torus, spheromaks, reversed pinches ...
  - Mirrors
  - Self-colliding beams (Migma proposed by Maglish in the early 70s).
- Inertial confinement
  - Laser & particle fusion
  - Non conventional approaches.

Basic facts and principles



How do electrons and ions gyrate?

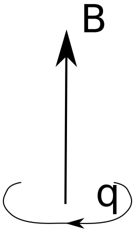
a) Both clockwise
b) Both counterclockwise
c) Electrons clockwise, ions counterclockwise
d) Electrons counterclockwise, ions clockwise

In addition to gyrating, charged particles also travel along magnetic field lines.

Adiabatic invariants

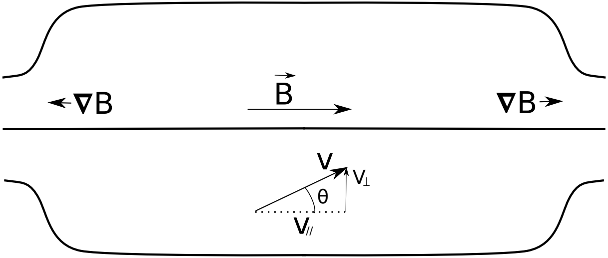
- From classical mechanics, $\oint \vec{p} \cdot d\vec{q}$ is an adiabatic invariant, that is, this quantity remains nearly constant when the system changes 'slowly' compared to an orbit period.
- Here $\vec{p}$ and $\vec{q}$ are canonical momentum and coordinates respectively.
- In Cartesian coordinates, for a non relativistic particle, $\vec{p} = m\vec{v} + q\vec{A}$.
- In plasma, there are three adiabatic invariants:

First adiabatic invariant



- $\vec{p} \simeq m\vec{v} - \frac{q}{2}\vec{r} \times \vec{B}$
- $\vec{r} = \rho_L \left[\cos(\Omega t)\hat{x} - \sin(\Omega t)\hat{y}\right]$
- $\vec{v} = v_\perp(-\sin(\Omega t)\hat{x} - \cos(\Omega t)\hat{y})$
  where $v_\perp = \Omega\rho_L$
  $\rightarrow \mu = \frac{mv_\perp^2}{2B}$ corresponds to the magnetic moment of the gyrating particle.

The mirror force



- If $\vec{E} = 0$, $\frac{mv^2}{2} = \frac{mv_\parallel^2}{2} + \mu B = \text{constant}$.
  $\rightarrow |v_\parallel|$ decreases when approaching regions where B is large $\rightarrow$ Trapping.
- $\rightarrow$ Trapping when $\mu B_{max} = \frac{mv_\perp^2}{2B_{min}} B_{max} \geq mv^2/2$ or $\theta > \theta_L$.
- $\rightarrow$ loss cone angle $\theta_L = \mathrm{asin}\left(\sqrt{\frac{B_{min}}{B_{max}}}\right)$
- $\vec{F}_m = -\mu \nabla B$

Second invariant

- Periodic motion: Bouncing back and forth of a particle along field lines, between two (high B) turning points.
- In this case, $\oint \vec{A} \cdot d\vec{r}$ cancels and only the kinetic part of the momentum contributes
- $\rightarrow J = \oint \vec{v}_{||} \cdot d\vec{r}$

Particle drifts

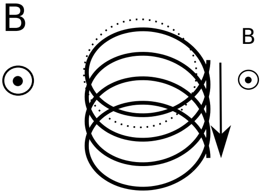
- In a presence of a straight, uniform and constant magnetic field, the motion of a charged particle exhibits a helix.
- When an electric field is present, however, the motion describes a cycloid with a drift velocity $v_{E\times B} = \frac{\vec{E}\times\vec{B}}{B^2}$.
- In the frame moving with that velocity, the electric field vanishes and the particle motion again describes a helix.
- The situation is more complex when there are other forces, when the fields ($\vec{B}$ or $\vec{E}$) are not uniform or when they vary in time.
- In the presence of a force field $\vec{F}$, the drift is given by

$$\vec{v}_D = \frac{\vec{F}\times\vec{B}}{qB^2}$$

Grad-B drift

Grad-B drift:

$$\vec{v}_{\nabla B} = -\frac{\mu}{qB^2}\nabla B \times \vec{B}$$
$$= -\frac{mv_\perp^2}{2qB^3}\nabla B \times \vec{B}$$



- Particles of different charges drift in different directions.
- $\rightarrow$ currents.

Curvature drift

- Here the force is the 'centrifugal' force:

$$\vec{F}_c = -mv_{\parallel}^2 \hat{b} \cdot \nabla \hat{b}$$

- The associated drift is

$$\vec{v}_c = -\frac{mv_{\parallel}^2}{qB^2} \left(\hat{b} \cdot \nabla \hat{b} \right) \times \vec{B}.$$

- For a low β plasma; that is, when $\beta = \frac{nkT}{B^2/\mu_0} \ll 1$, this drift can be written in almost the same form as the curvature drift:

$$\vec{v}_c \simeq -\frac{mv_{\parallel}^2}{qB^3} \nabla B \times \vec{B}$$

Remarks about drifts

- Only the $\vec{E} \times \vec{B}$ drift does not lead to a current. For all other drifts, particles of different mass or charge are affected differently.
- There are other forces and associated drifts (e.g., associated with gravity).
  Those can be important in astrophysical plasma, but they are negligible in tokamaks.
- Drifts are only perturbations to particle trajectories moving along circular (helicoidal) orbits to lowest order.
- The small parameter in the perturbation expansion is effectively $\frac{1}{B}$, or more precisely $\frac{mv_\perp}{qBL}$.
- All drifts seen so far are first order in this small parameter.

Polarization drift

- The polarization drift

$$\vec{v}_p = \frac{m\dot{\vec{E}}}{B^2}$$

is the exception. It is second order in $1/B$.

- $\vec{v}_p$ is important in tokamak equilibrium and drift waves because of the large ion to electron mass ratio. In practice it only affects ions and it is negligible for electrons.
- Because of the polarization drift, plasma behaves as a medium with high dielectric constant (polarizability) in the direction perpendicular to $\vec{B}$ (See Kulsrud's "Plasma Physics for Astrophysics").
- If $1/B$ is not a small parameter, then the whole concept of drifts breads down, and particle trajectories must be integrated in detail.

Third adiabatic invariant Φ

- In certain magnetic field configurations (particles ∇B or curvature-drift across field lines as they gyrate around and bounce back and forth along magnetic field lines.
- The resulting quasi-periodic drift motion $\rightarrow$ Third adiabatic invariant.

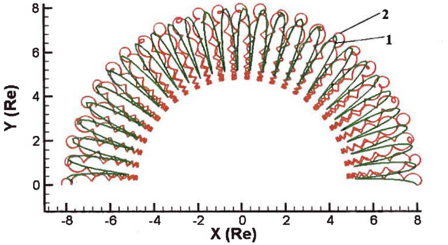


Figure 2.3: Projection in the XY plane of calculated trajectory of trapped protons using both guiding centre equations (label 1) and full Lorentz equations of motion (label 2) in a dipole field. In both cases, the proton is located initially at x=8R_e, y=0, with initial energy 1MeV and a pitch angle of 30°.

(Ding Li, MSc thesis UofA 2006)

Physical meaning of Φ

$$\Phi = \oint d\vec{r} \cdot \left(\underbrace{m\vec{v}}_{\text{small}} + q\vec{A} \right)$$
$$\propto \atop \sim \quad \oint d\vec{r} \cdot \vec{A} = \iint da\, \nabla \times \vec{A} \cdot \hat{n} = \iint da\, \vec{B} \cdot \hat{n}$$

$\rightarrow \Phi$ represents the magnetic flux enclosed in in the quasi-periodic drift motion.

Occurrence:

- Linear confinement devices such as mirrors.
- Planetary and astrophysical$\sim$ dipole magnetic fields

Final remarks on invariants

- 'Invariants' are $\sim$ constant if the system changes slowly compared to a full revolution period.
- They are 'asymptotically' constant: They should be expressed as an asymptotic series (M Kruscal, J. Math. Phys. 3, 806-28 1962).
- μ is associated with the shortest revolution period. It is the 'best conserved' invariant.
- J is associated with a longer revolution period. It is involved in 1^{st} and 2^{nd} order Fermi acceleration.
- Φ is associated with the longest revolution period. It is the 'least well' conserved invariant.

How well is μ conserved?

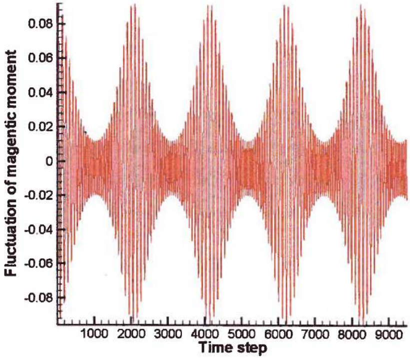


Figure 2.4: Variation in the magnetic moment calculated from a full Lorentz particle trajectory integration. The particle has the same initial conditions as in Figure 2.3.

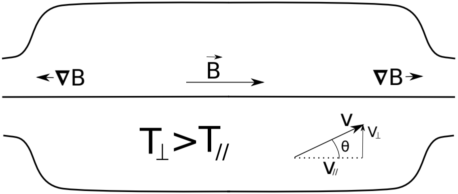
$$\Delta\mu = \frac{\mu - \mu_0}{\mu_0} \propto \epsilon = \frac{\rho}{l}. \tag{2.20}$$

(Ding Li, MSc thesis UofA 2006)

Magnetic confinement

- Charged particles 'stick' to magnetic field lines. They gyrate around and travel along $\vec{B}$.
- Magnetic confinement can be separated from material boundaries
- This is important considering the high temperature plasma needed to achieve fusion.
- Particles can drift slowly to nearby magnetic field lines. The magnetic field configuration must be such that these drifts effectively cancel.

Magnetic bottle: mirror machine



Fraction of velocity space in which particles are trapped:

$$\int_0^{\cos(\theta_L)} d\mu = \cos(\theta_L) \simeq 1 - \frac{B_{min}}{B_{max}}$$

Solution:

- Make sufficiently long mirrors.
- Make B_{max}/B_{min} "large enough".

Mirror machine: Problems

- Coulomb collisions, $\rightarrow$ diffusion of particles in the loss cone $\rightarrow$ loss of confinement.
- Increasing B_{max}/B_{min} doesn't help much because the confinement time scales logarithmically with (B_{max}/B_{min}): $\tau \propto \ln (B_{max}/B_{min})$.
- There are strong instabilities caused by velocity anisotropy $(T_{\perp} > T_{\parallel})$.

This concept was abandoned in the early 1980s.

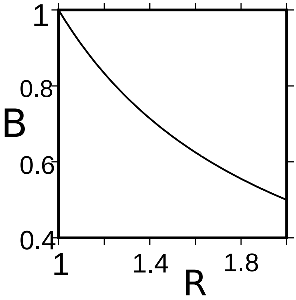
Magnetic Fusion Test Facility

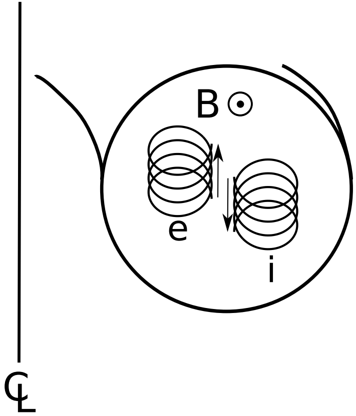
(http://www.energy.gov/articles/photo-week-mirror-fusion-test-facility)

Confinement in a torus ☺

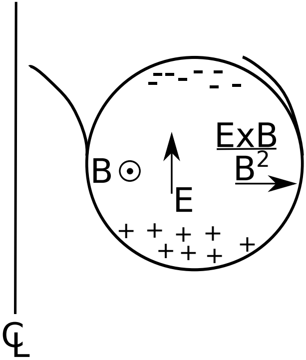
- Field lines trace closed flux surfaces
  $\rightarrow$ There are no 'ends' where particles can be lost.
- Relatively simple, especially in tokamaks where coil winding is topologically simple.
- Stability is well understood theoretically, computationally and empirically.
- Classical/neoclassical transport is well understood.

Drifts



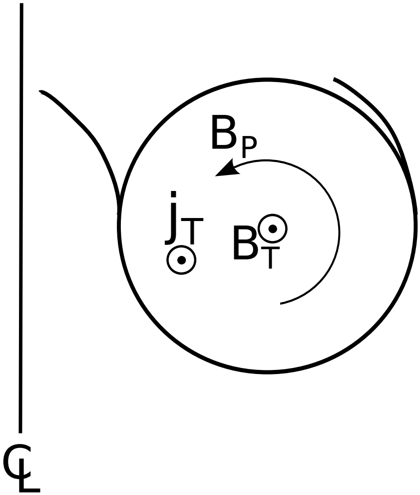


Loss of particles

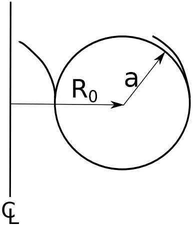


- In a finite density plasma, drifts $\rightarrow$ electric field $\rightarrow \vec{E} \times \vec{B}$ drifts.
- $E = Jt = nev_D t \rightarrow v_{\vec{E}\times\vec{B}} = \frac{nev_D t}{\epsilon_0 B} \sim$ $\rightarrow$ displacement $= nev_D t^2/2B$
- Loss of particles.

Solution: rotational transform



Some definitions



- Minor radius: a
- Major radius: R
- Inverse aspect ratio: $\epsilon = \frac{a}{R}$
- Rotational transform: ι
- Safety or 'q' factor: $q = \frac{2\pi}{\iota}$

- Particle confinement time τ_p: Average time spent in the machine by a particle
- Energy confinement time τ_E: In the absence of external sources: $\frac{dE}{dt} = \frac{E_{plasma}}{\tau_E}$. At steady state: $E_{plasma} = P_{total} \times \tau_E$.
- Flux surface: toroidal surface generated by magnetic fields. They can be rational or irrational.
- Magnetic shear: $\Theta = dq/dr$

Some difficulties ☹

- Instabilities remain challenging, such as ‘tearing instabilities’ having to do with reconnection.
- Particle and energy transport is usually not neoclassical. There are empirical scaling laws based on existing machines, but it is difficult to make projections for significantly larger future machines.
- Transport is typically governed by turbulence, and that is a very difficult problem.
- There are many difficult technical and engineering problems not yet resolved.

Some technical issues

- First wall and plasma-material interaction
  - Neutron bombardment.
  - Erosion, sputtering and redeposition.
  - Disruption damage, particularly in the divertor.
  - Divertor physics.
  - Impurity transport and radiative losses.
- Fueling
  - Neutral beams would not penetrate deeply enough.
  - Pellet injection

More issues

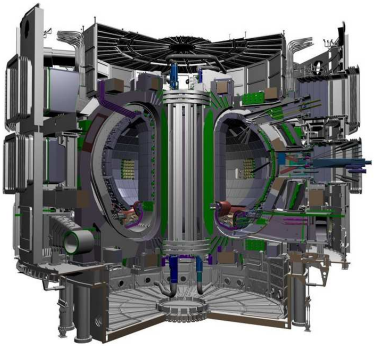
- Current drive
  - Inductive
  - Waves (lower hybrid)
  - Neutral beams
  - Bootstrap
- Recycling
  - Most of the fuel in the plasma will diffuse to the edge without burning.
  - It will have to be pumped out or recovered from the blanket and ‘recycled’ back in the reactor.
- He^4 ash buildup
- ...

Who participates?

- Many countries have now developed research programs in fusion and in tokamak physics.
- Fusion is much more difficult that anyone had anticipated some 60 years ago.
- Front line research presently focuses on a large international project: ITER
- Domestic Agencies in ITER: China, EU, India, Japan, Korea, Russia, US

ITER

- 2019: Complete tokamak assembly.
  Begin commissioning
- 2020: First plasma
- 2027: Start D-T operations
- Construction cost: $\sim$ 13 B €.



(http://www.iter.org/mach)

Will it work? If so when?

- ITER is an experimental reactor.
  It is not meant as a demonstration reactor.
- It should be followed by a demo reactor.
- This would eventually be followed by the first commercial reactors.
- Future projects will likely involve large international collaborations.
- They will take many years to be negotiated and decided.