



Introduction to single-pass FELs for UV – X-ray production

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Outlook

- Motivations
- Radiation emission in undulator
- Self-Amplified Spontaneous Emission (SASE) FEL
- FEL requirements for electron beam
- FEL design from scratch
- Examples
- Summary

An ideal light source should provide...

High resolution at small spatial scales

Most of the photons at the same wavelength

narrow bandwidth

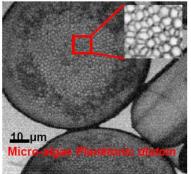
Stroboscopic picture of chimical processes

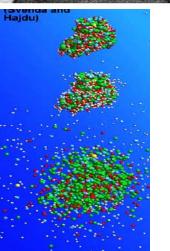
Large statistics in single-shot

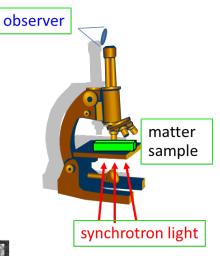
→ large number of photons per pulse

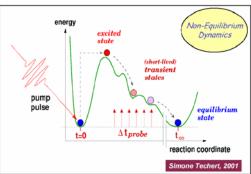
Large statistics in multi-shot

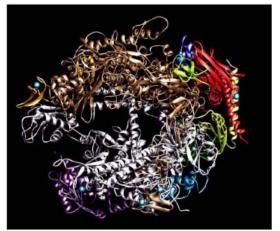
 \rightarrow high repetition rate







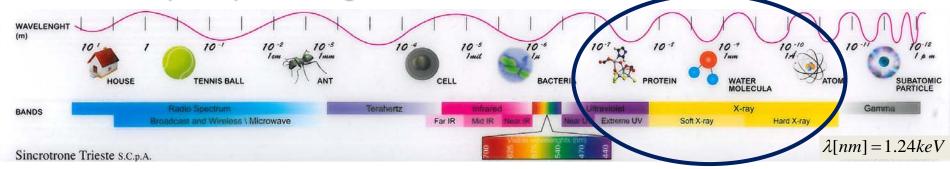




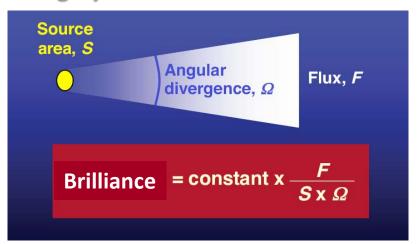
An ideal light source should be...

1. Tunable in (Short) Wavelength

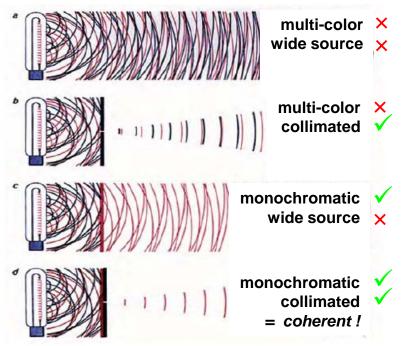
Range of interest here



2. Highly Brilliant



3. Fully Coherent



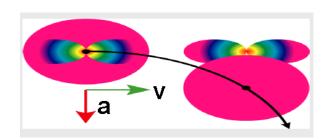
Courtesy of A. Schawlow, Stanford.

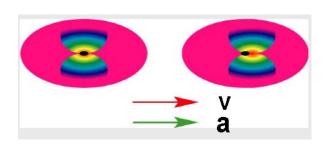
Radiation emission of a charged particle

- ☐ A charged particle radiates under acceleration.
- Ligther particles such as electrons, radiate more than heavier ones such as protons, when subjected to the same force. Circular acceleration is more efficient than linear.

$$P_{circ}[kW] = \frac{2}{3} \frac{e^2}{c^3} \frac{\gamma^2}{m_0^2} \left| \frac{d\vec{p}}{dt} \right|^2 = 88.46 \frac{E^4 [GeV] \cdot I[A]}{R[m]}$$

$$P_{lin} = \frac{P_{circ}}{\gamma^2}$$





- Crentz-boosted lab. frame

 Orbit

 Orbit

 Orbit

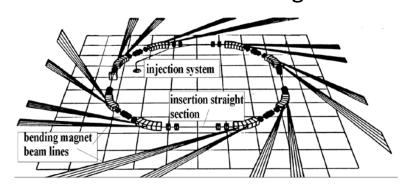
 (a)

 Lorentz-boosted lab. frame

 | According to the content of the content of
- Circular acceleration can be provided by dipole magnetic field: synchrotron light sources.
- Linear acceleration can be provided by longitudinal electric field: RF structures.

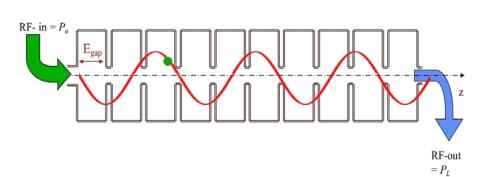
Why a LINAC-driven light source

e-beams in **synchrotron light sources** (SLS) reach **equilibrium properties** that are typically far from providing short pulses, high intensity, and narrow bandwidth at short wavelengths.





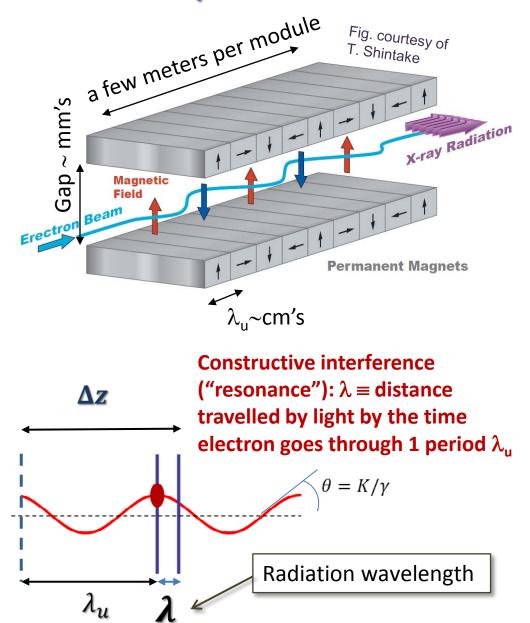
☐ An electron radiofrequency linear accelerator (RF e-LINAC) can be used to overcome the SLS equilibrium dynamics and to "shape" the e-beam as desired.



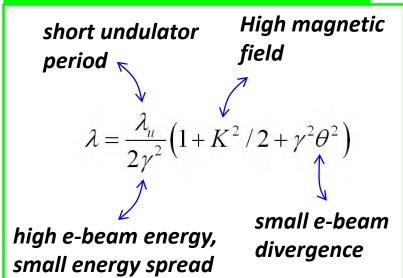


However, a more efficient **radiating process** is still needed to surpass the SLS's brilliance level...

Undulator Spontaneous Emission



Undulator resonance wavelength:



Undulator strength parameter:

$$K = 0.934 \lambda_u [\text{cm}] B_{max} [\text{T}] \approx 1 \div 10$$

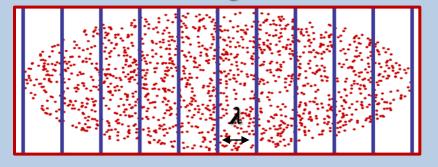
Incoherent vs. Coherent Emission

no. photons emitted by 1 electron through N_u undulator periods in $\sim 1/N_u$ bandwidth and $\sim 1/(\gamma^2 N_u)$ solid angle

$$N_{ph} \sim \pi \alpha \frac{K^2}{1 + K^2/2}$$

$$\alpha=e^2/\hbar c$$
 = 1/137 (cgs)

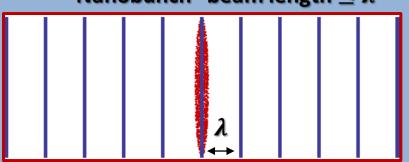
Bunch length $\gg \lambda$



Linear in no. of electrons/bunch
$$\sqrt[k]{N_{ph} \sim N_e \pi \alpha} \frac{K^2}{1 + K^2/2}$$

Incoherent emission

"Nanobunch" beam length $\leq \lambda$



Quadratic in no. of electrons/bunch

$$N_{ph} \sim \frac{\sqrt{k^2}}{N_e^2 \pi \alpha} \frac{K^2}{1 + K^2/2}$$

Fully coherent emission

How do micro/nano-bunches form?

- 1. Electrons initially radiate 'undulator spontaneous emission'. Their longitudinal density distribution is governed by shot noise.
- 2. The electron transverse velocity couples to the photons' transverse electric field.
- 3. To amplify in intensity, the light should *overlap* with the electrons and out race them by one optical wavelength (2π in phase) after they have traveled one full cycle, *i.e.* $\lambda_{\rm u}$.

$$\frac{\lambda_u + \lambda_s}{c} = \frac{\lambda_u}{v_z} \quad \Longleftrightarrow \quad \lambda_s = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} \right)$$

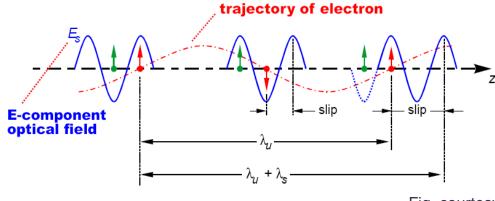


Fig. courtesy of R. Bakker

- 4. The electrons are modulated in energy at wavelength λ . Their path-length dependence on the energy (intrinsic energy-dispersion in the undulator is $2N_u\lambda_u$) translates the energy modulation into **density modulation** ("bunching") at the same resonance wavelength.
- 5. The micro/nano-bunched beam emit coherently \rightarrow Self-Amplified Spontaneous Emission (SASE FEL).

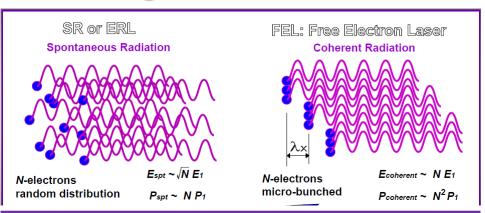


FEL resonance condition = undulator radiation constructive interference.

This allows the amplification of the "undulator signal"

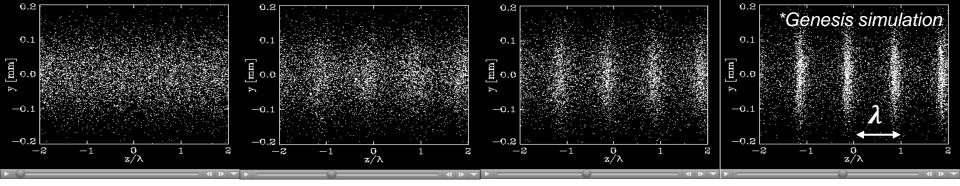


Bunching

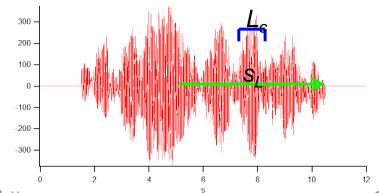


- If the electrons are independently radiating, then the phase of their electric field are random with respect to one another, like in SLS.
- If the electrons are in lock synch and radiate coherently, like in FELs, one can get an enormous gain (N) in power emitted.

Snap shots of small portion of e-beam developing bunching along the undulator line*



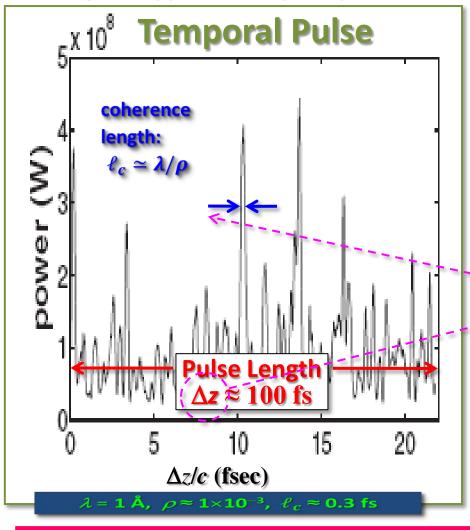
The FEL pulse consists of several **coherent** regions (spikes, L_c) randomly distributed over the e-bunch length. The photons slip over the electrons (slippage, s_L) "connecting" multiple spikes.

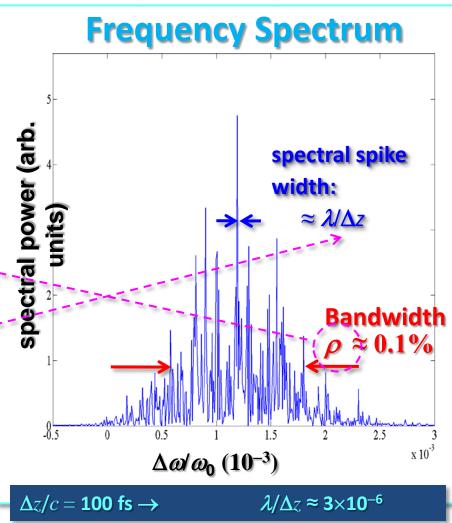


Temporal & spectral radiation pulse structure

spikes appear in temporal pulse





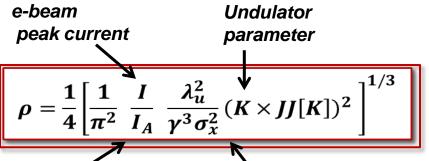


• Longitudinally radiation is fully coherent over ℓ_c , typically shorter than bunch length

• What is ρ ?

1-D Model for SASE FEL

• 'Cold beam' approx.: zero-emittance, zero-energy spread. Infinitely wide beam with uniform transverse density (no radiation diffraction effects).



• Pierce parameter ρ . The jack of all trades of 1D FEL theory. Typical values $\rho \lesssim 10^{-3}$

Alfven curren
$$I_A \simeq 17kA$$

Transverse e-beam rms beam size

$$\ell_c = \frac{1}{4\pi} \frac{\lambda}{\rho}$$

• Cooperation length. The length over which electrons within bunch can "communicate" with each other (i.e. how far ahead the radiation emitted by an electron goes by the time it travels through L_a .)

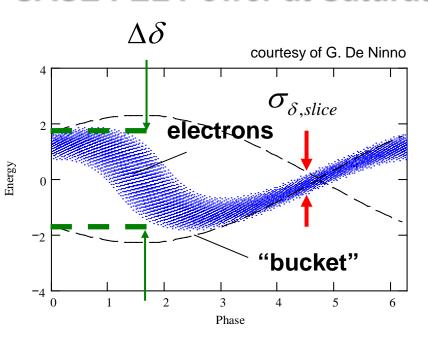
$$L_g = \frac{1}{4\pi\sqrt{3}} \frac{\lambda_u}{\rho}$$

• The FEL power gain length is inversely proportional to ρ . The smaller the more efficient the FEL process, the sooner we achieve saturation, the shortest the undulator, the less \$\$\$ we need to spend.

$$P = P_0 e^{s/L_g}$$

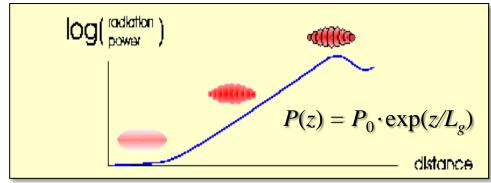
 Radiation power grows exponentially along the undulator (typical behavior for instability-driven processes) until saturation.

SASE FEL Power at Saturation



At saturation, the beam energy spread reaches its maximum:

- → the longitudinal phase space becomes folded;
- → electrons gain energy from radiation.



$$P_s \sim \rho P_b$$

• Radiation power at saturation is proportional to ρ and e-beam power: $P_h = E_h I/e$.

$$\frac{\sigma_{E,sat}}{E} \sim \rho$$

 Beam energy spread increases along the undulator because of interaction with radiation. When it becomes too large, electrons start falling off resonance, saturation follows.

$$L_s \sim \frac{\lambda_u}{\rho}$$

FEL power saturation length. About $\sim 4\pi \sqrt{3}L_g \sim 20 L_g$ gain length. This sets the scale for the undulator length

e-Beam Requirements for Lasing

$$\rho = \frac{1}{4} \left[\frac{1}{\pi^2} \frac{I}{I_A} \frac{\lambda_u^2}{\gamma^3 \sigma_x^2} (K \times JJ[K])^2 \right]^{1/3}$$

1-D model \rightarrow FEL performance benefits from large $\rho \rightarrow$ beam high current, small transverse emittance.

$$\frac{Q}{c\Delta t} > 0.1 \ kA$$

• e-beam peak current should be high (typically from 100s A to kA level) that is a high charge and/or short bunches.

$$arepsilon_{\perp} \lesssim rac{\lambda}{4\pi}$$

e-beam transverse emittance should be on the order of, or smaller than, the radiation emittance (diffraction limit) $\varepsilon_r = \frac{\lambda}{4\pi}$.

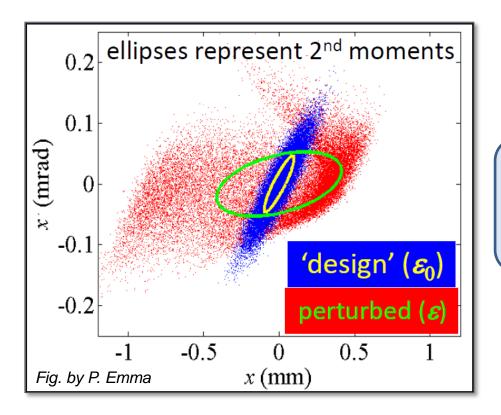
$$\lambda = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} \right)$$

FEL wavelength → resonance condition for on-energy electrons.

$$\frac{\sigma_{E,und}}{E} < \rho$$

Relative energy spread at the undulator entrance should be small. Electrons with energy too different from nominal slip off the FEL energy-resonance and do not contribute to lasing.

Introduction to beam 2-D emittance



$$\langle x^2 \rangle \equiv \int x^2 f(x, x') dx dx'$$

f = beam density distribution function normalized
to unity

The beam rms ellipse is defined by the 2D curve in the (x, x') plane:

$$\varepsilon_{x} = \gamma_{x}^{*} x^{2} + 2\alpha_{x}^{*} x x' + \beta_{x}^{*} x'^{2}$$

$$\sim \text{ellipse} \qquad \sim \text{ellipse} \qquad \sim \text{ellipse}$$

$$\text{size in } x' \qquad \text{orientation,} \qquad \text{size in } x$$

$$x/x'$$

- Geometrical meaning of rms emittance: area $A = \pi \epsilon_x$ of the beam rms ellipse
- Physical meaning of rms emittance: spread of the particles in the phase space (x, x')
- Brightness is best expressed in terms of "normalized" emittances. These are linearly invariant in the presence of acceleration: $\varepsilon_{nx,y} = \gamma \beta \varepsilon_{x,y} \simeq \gamma \varepsilon_{x,y}$ $\varepsilon_{nz} = \sigma_z \sigma_{E,uncor}$

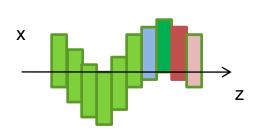
Slice vs. projected emittance

- To function effectively, FELs need beams meeting minimum brightness requirements.
- A concept of 4D or 5D brightness can be also useful:

$$B_4 = rac{Q}{arepsilon_{nx} arepsilon_{ny}}$$
 or $B_5 = rac{I}{arepsilon_{nx} arepsilon_{ny}}$ Beam peak current

- Slice vs. projected transverse emittance:
 - For lasing, what counts primarily is the rms emittance of the particles within a longitudinal beam *slice* on the order of the cooperation length (where the electrons 'talk' to each other) or the slippage length.

 However we should not let the projected emittance grow too much or else individual slices will not be all properly matched to the intended e-beam optics in the undulator.



Snapshot of beam in x/z plane (various slices highlighted)

Individual slices may have the same (slice) emittance but if the slice rms ellipses are not concentric, the emittance of the whole beam is larger (projected emittance)

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e-Beam 3-D effects (M. Xie's model)

We come up with a figure of merit that captures all the desirable e-beam properties at once - high peak current, small transverse emittance, small energy spread: e-beam 6-D brightness (energy-normalized).

$$B_{6D} = \frac{N}{\varepsilon_{nx} \varepsilon_{ny} \varepsilon_{nz}}$$
 No. particles/bunch

No. particles/bunch

No. particles/bunch

No. particles/bunch

M. Xie (mid $\sim 90s$) gave a parametrization of L_a based on numerical solutions of 3-D theory: very handy, used extensively for FEL design optimization.

$$L_{g} = L_{g0} \big[1 + \Lambda(X_{\delta}, X_{d}, X_{\varepsilon}) \big] \qquad X_{\delta} = \frac{4\pi\sigma_{\delta}}{\lambda_{u}} L_{g0}$$
3D-theory approximation to this function
$$X_{d} = \frac{\lambda}{4\pi\sigma_{r}^{2}} L_{g0}$$

$$X_{d} = \frac{\lambda}{4\pi\sigma_{r}^{2}} L_{g0}$$

$$X_{d} = \frac{\lambda}{4\pi\sigma_{r}^{2}} L_{g0}$$

$$X_{e} = \frac{4\pi\epsilon_{1}}{\beta_{twige}\lambda} L_{g0}$$

- Scaled energy $X_{\delta} = \frac{4\pi\sigma_{\delta}}{\lambda_{co}} L_{g0}$ Scaled spread
- $X_d = \frac{\lambda}{4\pi\sigma_r^2} L_{g0}$ Scaled transverse size
- $X_{\varepsilon} = \frac{4\pi \varepsilon_{\perp}}{\beta_{twiss} \lambda} L_{g0}$ Scaled transverse emittance
- ✓ 1D limit recovered when X_{δ} , X_{d} , X_{ε} → 0 while keeping the beam charge density constant
- \checkmark Lot of physics goes into L_a . For accurate determination do numerical simulations.

First estimate of parameters for FEL

design

- Science case generally drives the specifications for the FEL output:
 - Shortest wavelength of interest, say $\lambda \equiv 1$ nm.
 - Wavelength tunability, say $\lambda < 5$ nm.
 - No. photons per pulse, say >10¹² in the full λ -range.
 - Possibly, high repetition rate and/or multiple, independent beamlines working simultaneously.
- \Box Generally the challenge is to reach short radiation wavelength λ :
 - Shorter undulator period λ_u (min. value set by available technology and FEL performance)
 - Larger e-beam energy v (max value set by

Remind:

$$\lambda = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} \right)$$

Step 1: first rough assessment of needed e-beam energy (we will go back to it). Take $\lambda_u = 2cm$ and $K \sim 1 \Rightarrow mc^2 \gamma = mc^2 \sqrt{\frac{\lambda_u}{2\lambda}} (1 + \frac{K^2}{2}) \sim 2GeV$ (energy could get higher if we go with higher K to increase radiation output).

Choice of undulator gap

- lacktriangle λ -tuning range (i.e. range of radiation spectrum that can be generated once the undulator is installed and λ_u is fixed):
 - Vary beam energy (can pose operational nuisances; not practical if same linac feeds multiple FELs, operating at the same time and targeting different radiation wavelengths)
 - Vary undulator parameter K (i.e. magnetic field). Tuning range depends on undulator technology and requirements on the minimum undulator aperture (or 'gap')
- **Step 2**: independent beamline tunability \rightarrow we cannot vary energy to vary λ \rightarrow we have to adjust the undulator gap.

 **Remind:
 - Go with mainstream, well tested "Hybrid PM undulators"
 - Ask the magnet designer for magnetic field model:

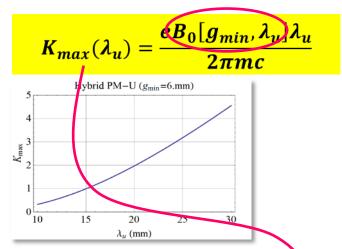
$$B_0[g, \lambda_u] = 4.22[T] \exp(-5.08 \times \left(\frac{g}{\lambda_u}\right) + 1.54 \times \left(\frac{g}{\lambda_u}\right)^2)$$

- Choose aggressive low-gap, out-of-vacuum undulator gap: $g_{min} = 6 \text{ mm}$
- g_{min} gives K_{max} for any fixed λ_u

 $K = \frac{eB_0\lambda_u}{2\pi mc}$

Choice of undulator parameter and beam energy

- Step 3 (a,b,c):
- a) Fix g_{min} = 6mm and compute $K_{max}(\lambda_u)$:

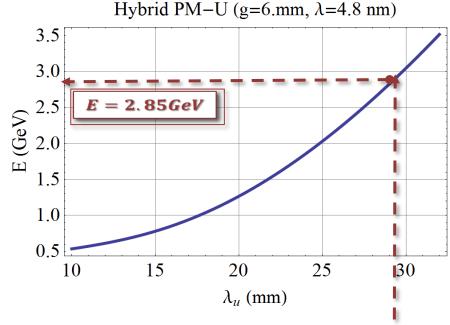


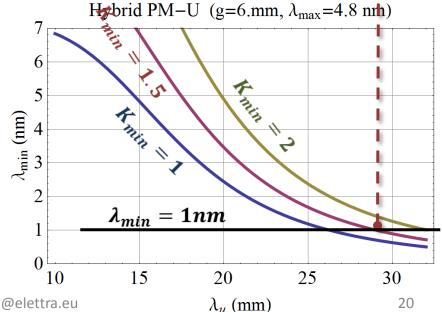
b) Fix λ_{max} = 4.8nm and compute $\gamma(\lambda_u)$:

$$mc^{2}\gamma = mc^{2}\sqrt{\frac{\lambda_{u}}{2\lambda_{max}}\left(1 + \frac{K(\lambda_{u})_{max}^{2}}{2}\right)}$$

c) Fix K_{min} >1 and and compute $\lambda_{min}(\lambda_u)$:

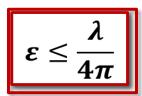
$$\lambda_{min} = \frac{\lambda_u}{2\gamma(\lambda_u)^2} \left(1 + \frac{K_{min}^2}{2}\right)$$





Choice of beam charge and emittance

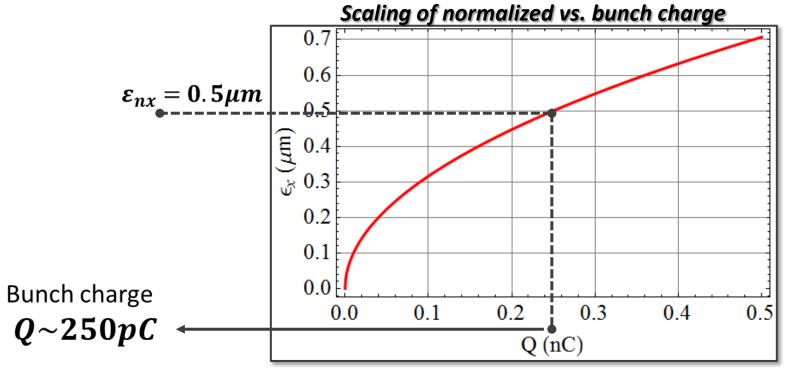
• Match e-beam emittance to radiation emittance at λ =1nm (most demanding wavelength) and E=2.85 GeV:



Normalized rms emittance
$$arepsilon_{nx}=\gammarac{\lambda}{4\pi}=5560 imesrac{10^{-9}m}{4\pi}=0.45\mu m$$

Call it $\varepsilon_{nx} = 0.5 \mu m$ (slightly larger emittance is OK).

• The minimum emittance is set buy Gun . Use $\sim \sqrt{Q}$ scaling law for emittance. (roughly fitting measurements of SLAC Gun):

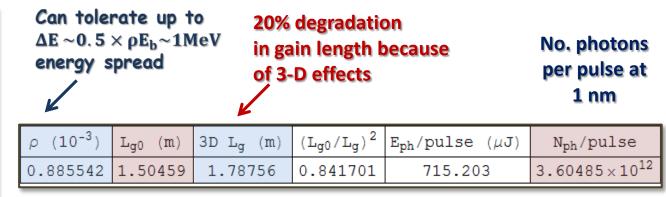


Estimate of FEL performance (1-D vs. 3-D model)

Beam/Machine Parameters

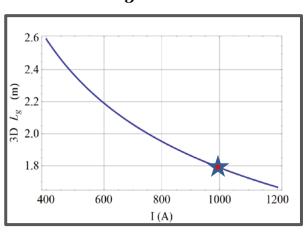
λ (nm)	1.
I (A)	1000
γ	5577.3
E (MeV)	2850.
λ_{u}	0_029
Twiss β (m)	12
$\epsilon_{\mathbf{n}\mathbf{x}}$ (μ m)	0.5
σ_{δ} (10 ⁻³)	0,0714286
σ_{E} (keV)	203.571
K	1.51345
Q (nC)	0.25

^{*} Not optimized

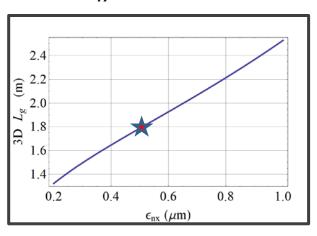


 Sensitivity study to independent variation of peak current, transverse emittance and energy spread:

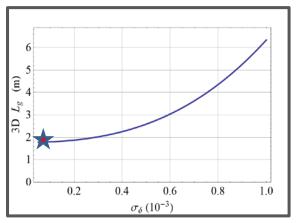
 $3D L_q$ vs. curr



 $3D L_a$ vs. rms emitted

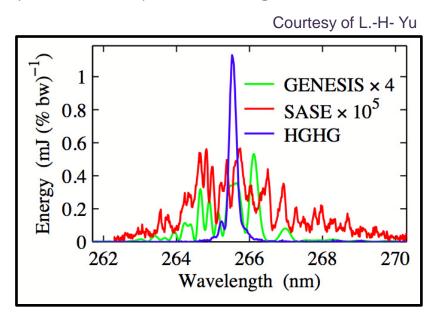


 $3D L_g$ vs. relative rms energy spread



Beyond SASE: (externally) seeded FELs

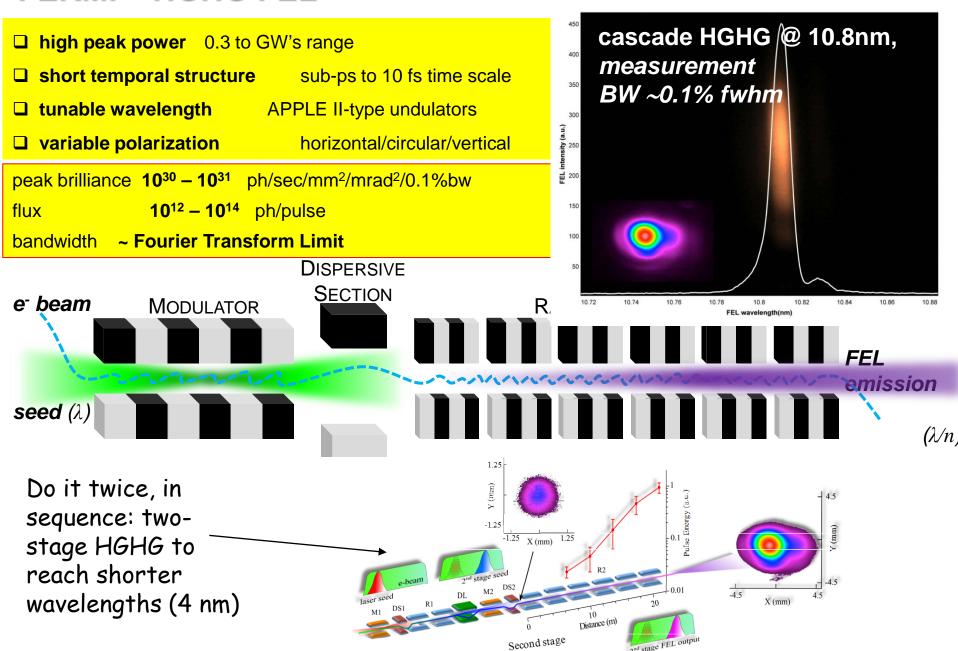
- Mainly to overcome the poor longitudinal coherence (relatively large spectral bandwidth) from SASE FEL, external seeding uses a fully (longitudinally) coherent radiation pulse to initiate the FEL process.
- Several schemes currently in use:
 - High Gain Harmonic Generation (HGHG)
 - Harmonic Cascades (two-stage HGHG, harmonic bunching)
 - Self-seeding
- Each FEL scheme has its own strong and weak points (e.g., limitation in wavelengths, intensity fluctuations, complexity, etc.). Chosen on the basis of specific requirements by users and/or complementarity to existing infrascture.
- Seeding poses additional demands on beam quality:
 - longitudinal coherence of radiation asks for a beam that has a long core with uniform profile;
 - uncorrelated energy spread should be low to reach high harmonics of the external laser (i.e., short wavelengths).



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0.4 0.6 0.8

FERMI - HGHG FEL

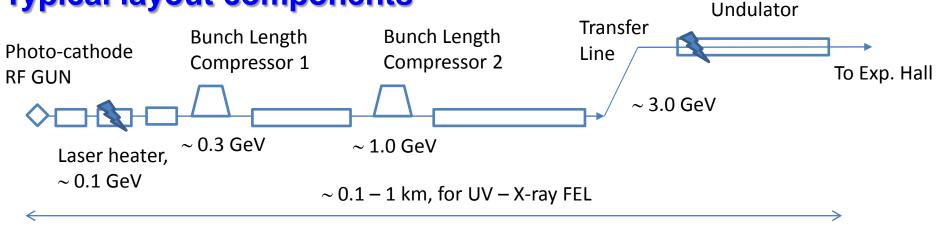


First Stage

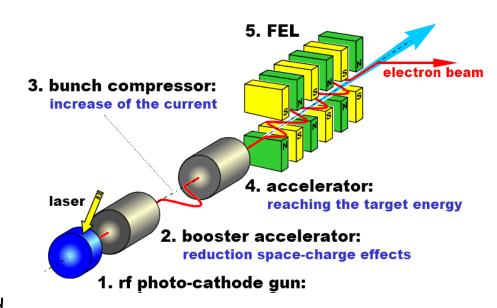
Recapitulating: what do we need for a UV - X-ray FEL

- ☐ High energy, monochromatic e-beam, possibly with high bunch rate
 - → Radiofrequency e-linac, normal- (NC) or super-conducting (SC)
 ~ 1 GeV for EUV, soft X-ray, ~ 10 GeV for Hard X-ray. Energy spread < 0.1%
- ☐ Small size, small angular divergence e-beam
 - High brightness e-sources, e.g. photo-injectors $\gamma \varepsilon_x \equiv \gamma \sigma_x \sigma_{x'} \leq 10 \mu m$ for EUV soft X-ray, $\leq 1 \mu m$ for hard X-ray
- ☐ Large number of particles in a short duration
 - Bunch length compressors, RF or magnetic
 I ~ kA over tens of femtoseconds
- ☐ Beam parameters have to be uniform over many cooperation lengths
 - → RF or magnetic manipulation of the longitudinal particle distribution Collective effects such as wakefields and coherent synchrotron radiation start playing a role here
- ☐ A low-gap, long enough undulator, with short magnetic period
 - Low-gap out-of-vacuum or in-vacuum undulator segments
 - $\lambda_{ij} \leq 10$ cm for EUV soft X-ray, ≤ 3 cm for hard X-ray. Total length $\sim 30-150$ m

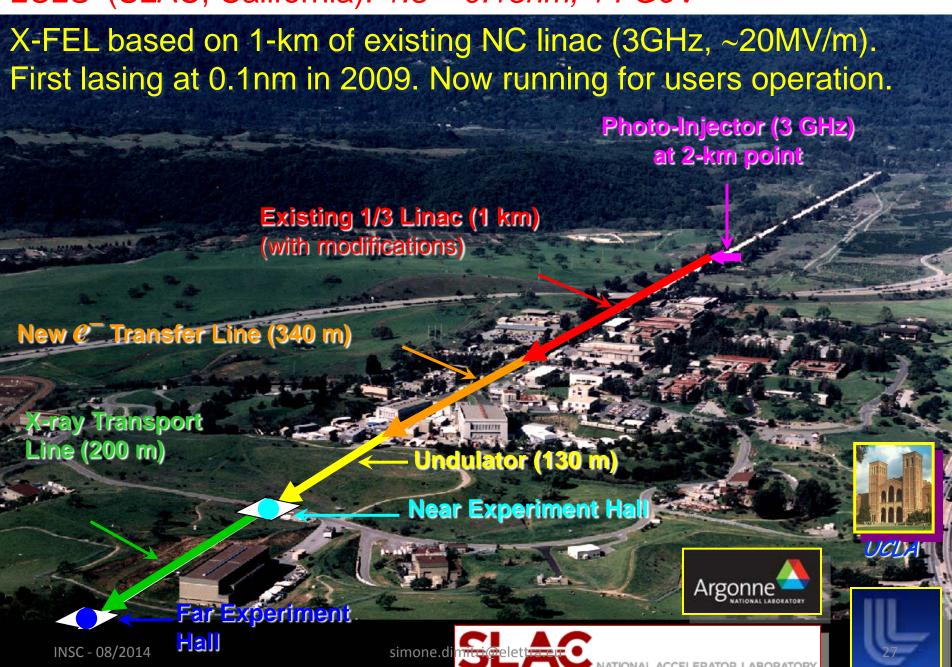
Typical layout components



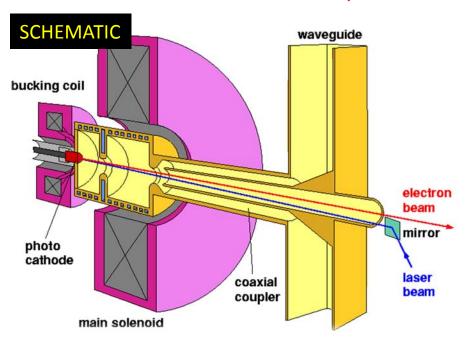
- PC RF Gun: ensures low transverse emittance beams
- Laser Heater: enlarges beam incoherent energy spread to damp instabilities
- Magnetic Compressors: increase the peak current
- RF linac: ensure the beam final energy and energy spread
- Transfer Line: switches the beam to different beamlines
- Magnetic focusing (not shown): ensures beam transport at reasonable transv. sizes
- Undulator: generates lasing till the desired output power

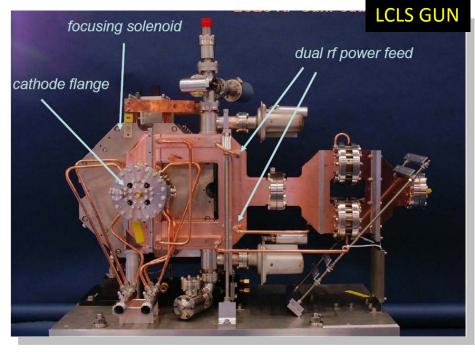


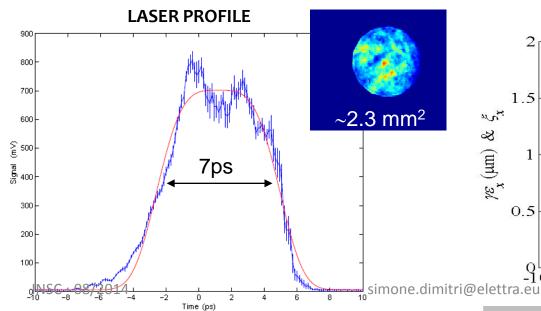
LCLS (SLAC, California): 1.5 - 0.15nm, 14 GeV

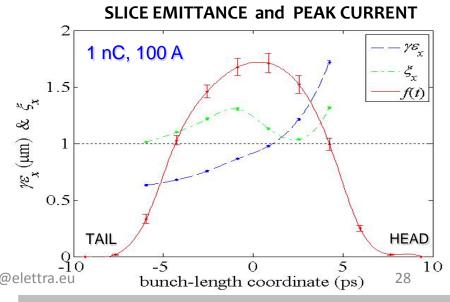


Figs. courtesy of D. Dowell

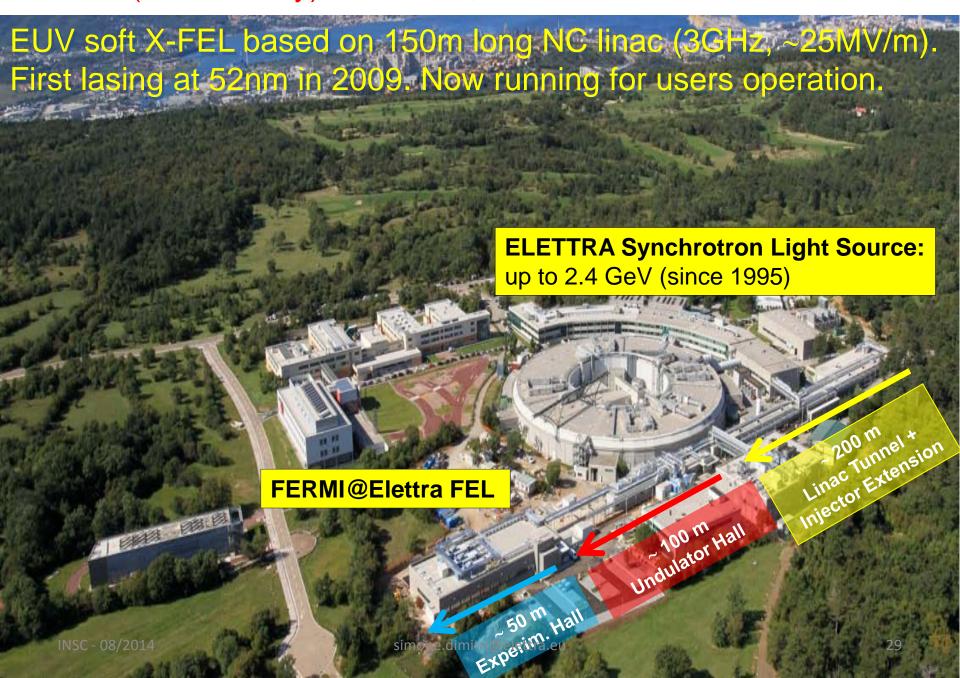




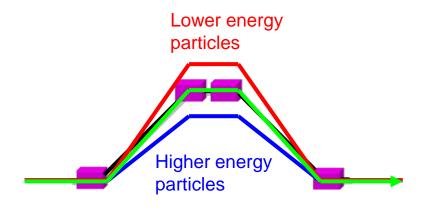




FERMI (Elettra, Italy): 100-4nm, 0.9-1.5 GeV

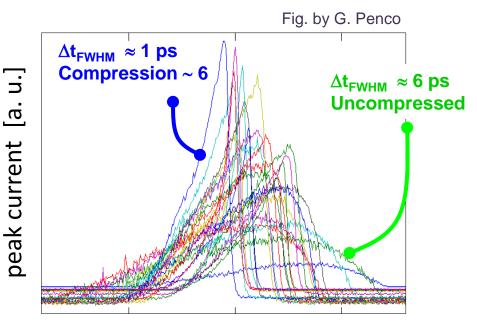


Magnetic bunch length compressor

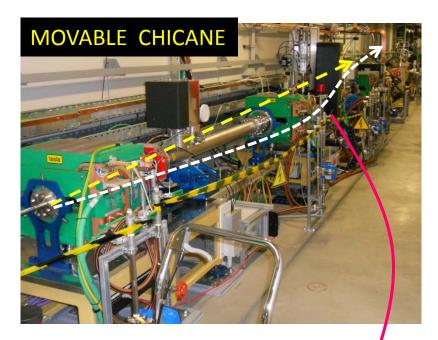


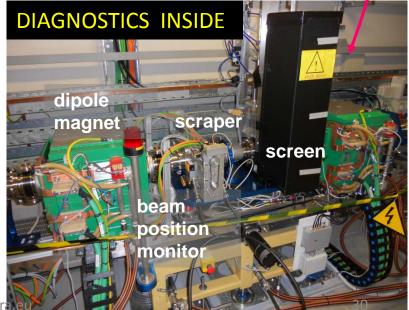
Path length difference (bunch shortening)

particles' energy difference



bunch duration [a. u.]

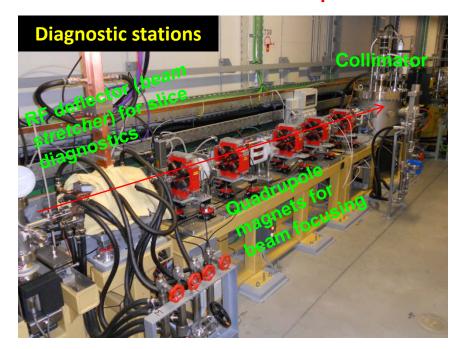


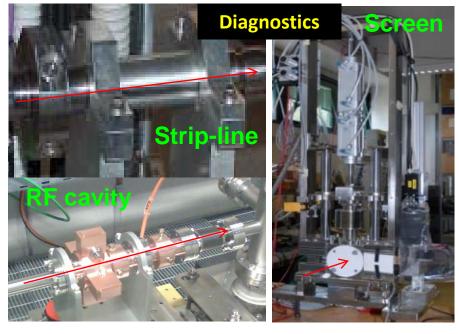


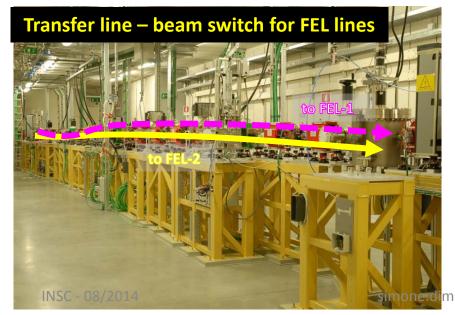
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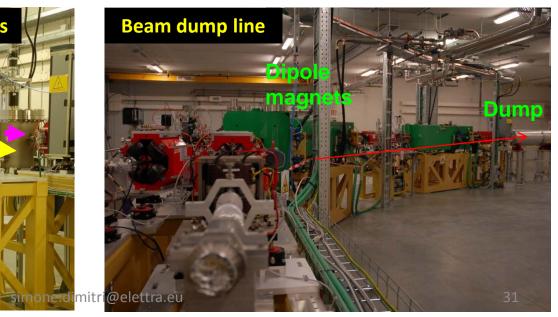
simone.dimitri@elettra

Other accelerator components





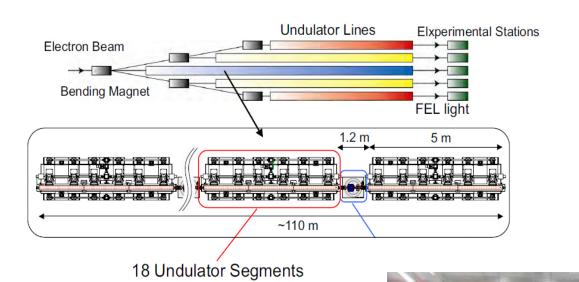




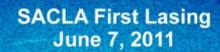
SACLA (Spring-8, Japan): <0.1nm, 8 GeV



In-vacuum undulator



Undulator Type		In-Vacuum Planer Undulator	
Active Length		5 m	
Undulator Period		18 mm	
Magnetic Circuit		Hybrid (NdFeB+Permendur)	
Peak Field	Maximum	1.31 T	
	Nominal	1.13 T	
K	Maximum	2.2	
	Nominal	1.9	
Gap	Minimum	3.5 mm	
	Nominal	4.5 mm	
Maximum Attractive Force		~ 6 ton	



Spontaneous radiation

X-ray laser (hv=10 keV)



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Summary highlights: what is an FEL?

- ☐ FELs are 4-th generation light sources:
 - SLS spectral range is extended to sub-Angstrom level
 - SLS brilliance is overwhelmed by ~9 orders of magnitude
 - Light pulse control: wavelength tunability, variable polarization, two-color pulses, pump-probe experiments
- ☐ Intense coherent radiation is emitted by "free" electrons in a tens of meter long undulator chain
- ☐ Electron beams are required to have very high 6-D brightness:
 - non-equilibrium dynamics in single-pass or recirculating linacs
 - round beams, transverse emittance smaller than ultimate SRs
 - up to 1000 times smaller longitudinal emittance than in SRs

Summary highlights: requirements on e-beam

FEL resonance condition → Look for short wavelength

short undulator period

$$\lambda = \frac{\lambda_u}{2\gamma^2} \left(1 + K^2 / 2 + \gamma^2 \theta^2 \right)$$

high e-beam energy, small energy spread small e-beam divergence

$$\square$$
 N large & short pulses $\Rightarrow I \sim kA$

$$\square$$
 e-/ γ transv. overlap $\Rightarrow \gamma \varepsilon \sim 1.0 \mu m$

$$\Box$$
 energy resonance $\Rightarrow \sigma_{\delta} < 0.1\%$

$$\square$$
 short $\lambda \Rightarrow E > 1$ GeV,

$$\lambda$$
 = 100 μ m \rightarrow ~ 15 MeV

$$\lambda$$
 = 10 nm \rightarrow ~1 GeV

$$\lambda$$
 = 1 nm $ightarrow$ \sim 3 GeV

$$\lambda = 1 \text{ Å} \rightarrow \sim 15 \text{ GeV}$$

$$P_{FEL}(z) \sim e^{z\frac{\rho_{FEL}}{\lambda_u}} \sim e^{z\left(\frac{I}{\sigma_\perp}\right)^{1/3}}$$

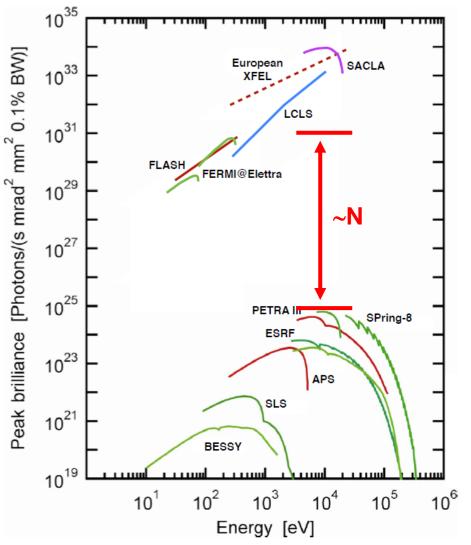
$$\varepsilon_n \le \frac{\gamma \lambda}{4\pi}$$

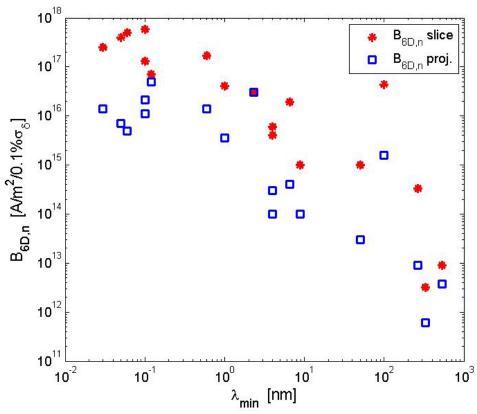
$$\sigma_{_{\!\delta}} <
ho_{_{FEL}}$$



High 6-D e-Beam Brightness

Summary highlights: FEL brilliance





The breakthrough of FEL brilliance is allowed by the cohrent emission.

FELs vs. Synchrotron Light Sources

	SLS	LINAC-FEL	Should be
Norm. emitt., γε [μm]	(10, 0.1)	0.1 – 1	≈ diffraction-limit
Energy spread, σ_{δ} [%]	0.1	0.01 - 0.1	<ρ _{FEL}
Bunch length, σ_{t} [ps]	10	0.1 - 1	Tunable in fs – ps range
Peak current, I [A]	10	1000	As high as possible
Repetition rate [Hz]	10 ⁸	$10^1 - 10^4$	As high as possible
Energy and intensity stability	10 ⁻⁵ - 10 ⁻⁶	10-2 - 10-3	As high as possible

• Storage rings are **complementary** to FELs as for λ -tunability, multiple-users access, stability and pulse rate.

Acknowledgements / References (not exhaustive)

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Thank You for Your kind attention

Questions and Comments are Welcome!