# *Short-Range Collective Effects*
## *in FEL linac drivers*

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# Outlook

- Motivations
- Longitudinal geometric wakefield,
  - in RF structure
- Transverse geometric wakefield,
  - in RF structure
  - in collimator
- Coherent synchrotron radiation
- Longitudinal Space Charge,
  - microbunching instability
  - laser heater

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# Why are collective effects important?

- Collective effects establish ***correlations*** between bunch **slices coordinates**, so that different slices may contribute differently to the FEL process, thereby affecting ***the FEL wavelength, bandwidth and intensity***.
- Correlations translate into transverse *projected emittance growth* and *enlarged energy spread*.

***Minimize collective effects to improve the FEL performance***

- Each bunch slice has to be ***transversally matched*** to the design optics for overlap with the radiation in the undulator, and set at the FEL ***resonance energy***.
- Control of e-beam slice parameters is inferred by the *measurement of e-beam projected values*: optics matching (i.e. Twiss parameters) in the transverse plane, energy chirp in the longitudinal.

***Make projected 6-D emittance as close as possible to its (average) slice value***

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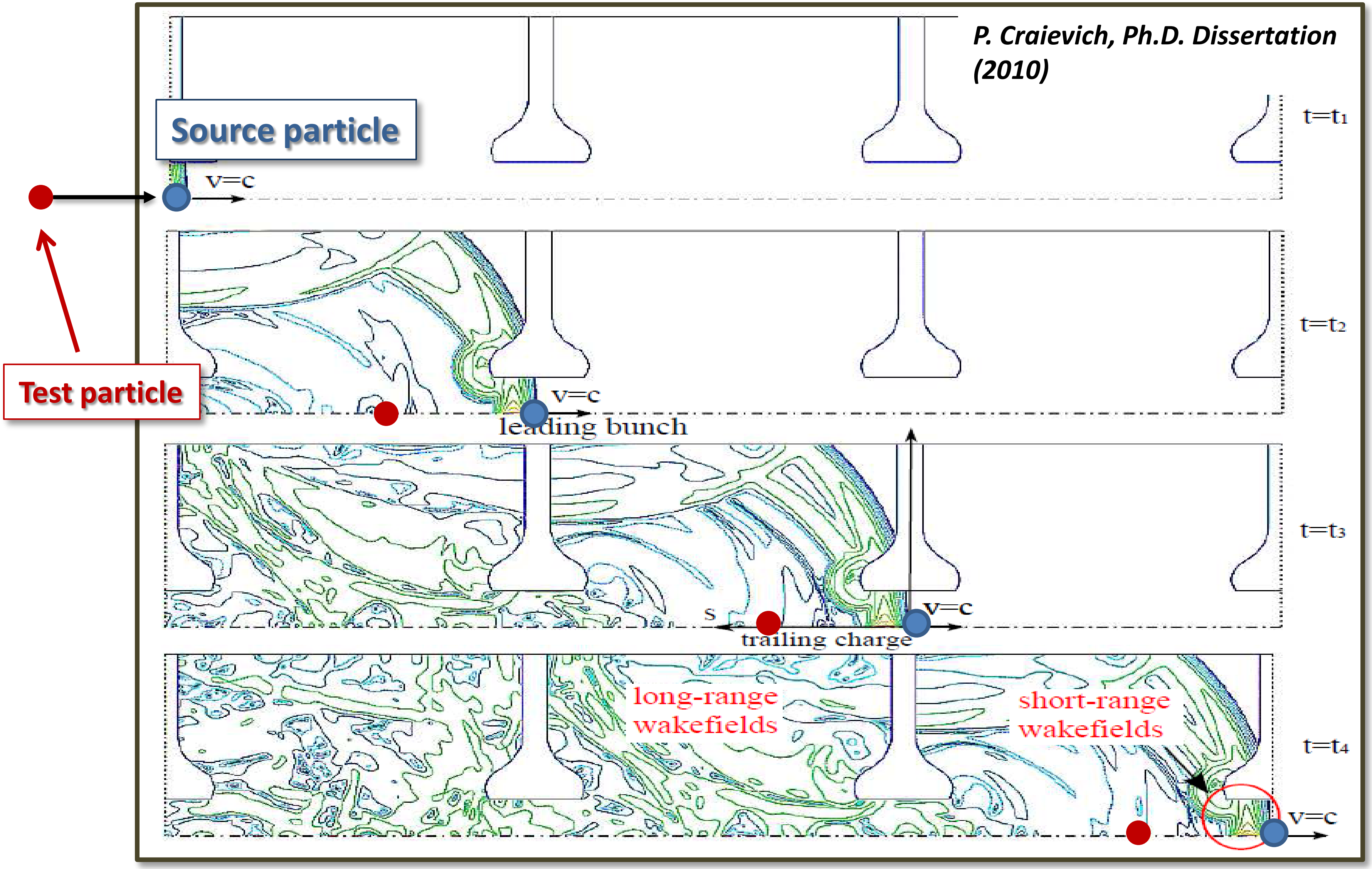
# Geometric wakefields in RF structure: the pictorial view

*Electric fields induced by passage of bunch through rf structure:*
*four snapshots in time*



*P. Craievich, Ph.D. Dissertation (2010)*

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# Wakefields in the time domain

- **How do wakefields affect the beam dynamics?**
  - kicking a particle **transversely:** $E_x, E_y, (\vec{v}_z \times \vec{B})_x, (\vec{v}_z \times \vec{B})_y$
  - Changing a particle **energy:** $E_z$

- **Wakefield are complicated functions of time and space** (see previous slide). To simplify the physical picture:
  1. Consider the e.m. field generated by a particle ('source'), at a given location and time. For now, uncouple electric and magnetic field, transverse and longitudinal components -> concept of **wake (Green) function.**
  2. Integrate over the beam charge distribution to calculate the effect of the entire bunch over a 'test' particle, at fixed distance w.r.t. the other 'sources' (ultra-relativisitc approx.) -> concept of **wake potential.**
  3. Integration of the wake potential over length of the structure washes out some of the complicated behavior.

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# Wake Function (also Green function or 'point-charge wakefield

## Longitudinal wake function

(space-integrated E-field over charge)

*E-field experienced by test particle (along direction of motion)*

$$W_L(|\Delta z|) = -\frac{1}{q_s}\int_0^L ds\, E_z(s\,, t = \frac{s+|\Delta z|}{c})$$

*Long. separation between source and test particles*

*"-" sign: a matter of convention*

*Source-particle charge*

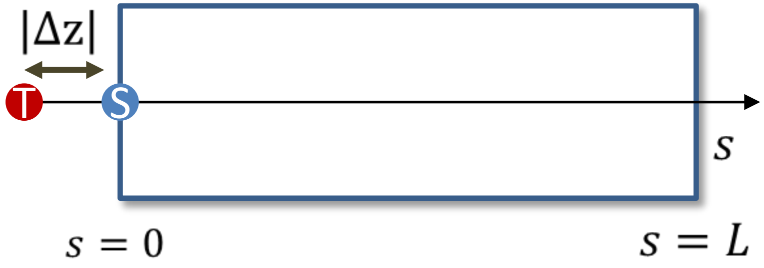
MKS units: $\frac{1}{C}\times m\times\frac{V}{m} = \frac{V}{C}$

Similarly: **transverse** wake potential (units: V/C)

$$W_\perp\left(\vec{r},\vec{r}',\Delta z\right) = -\frac{1}{q}\int_0^L ds\,[\vec{E_\perp} + c\left(\hat{x}\times\vec{B}\right)_\perp]$$

*Test particle trails behind at distance |Δz|*

*Source particle enters rf structure at time t=0*



- For a 'free' particle, $E_r = Cq/r^2$. Thus, $W_L \sim C/r$ and it only depends on the cavity **geometry** and boundary conditions (e.g., not on the charge distribution).
- What is $E_z$? It is the field generated by **image-charges** on the cavity inner surfaces (note: for relativ. particles the 'direct' electric field is boosted-forward in the direction of motion. This has nothing to do with wakefield).
- $W_L$ is **null** on particles **ahead of source**.
- You find often $w_L$ [V/C/m] ≡ $W_z$ / $L_c$

# Wake Potential (also 'bunch wakefield') and Energy Loss

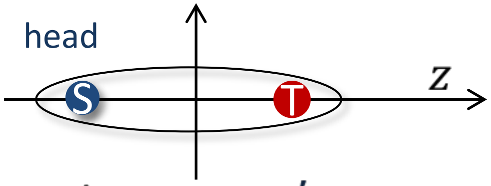
**Wake potential** is the **voltage drop** experienced by a **test particle** in the bunch, as a consequence of **all beam** particles **ahead** of it:

$$V_L(z) = Q\int_{-\infty}^{z} dz' W_L(z-z')\lambda(z')$$

MKS units: of $\mathrm{C} \times m \times \frac{V}{C} \times \frac{1}{m} = V$

Bunch charge $= Nq$

Coordinate system:



*Longitudinal bunch density (no. particles/m) normalized to unity* $\int dz'\,\lambda(z') = 1$

$\Delta z \equiv z - z'$

*Coordinate of test particle*

*Coordinate of source particle*

**Energy change** of **test particle** by source particle $(q_T = q_S = q)$

$$\Delta U_q = -q_T q_S W_L = -q^2 W_L \quad [V \times C = J \text{ or } eV]$$

Note on sign convention: $w_z > 0$ means energy loss i.e. $\Delta U<0$

**Energy change** of **test particle** by **entire bunch (ahead)**:

$$\Delta U_{Nq}(z) = -qV_L(z) = -Nq^2 \int_{-\infty}^{z} dz'\, W_L(z-z')\,\lambda(z') \quad [J \text{ or } eV]$$

Total energy loss of & by bunch:

$$U_{tot} = \int U(z)\,\lambda(z)dz$$

$$\boxed{k_l = \frac{U_{tot}}{Nq^2}}$$

- $k_l$ is called "***loss factor***"
- The normalization to $q^2$ makes it only dependent on the cavity **geometry**

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# Steady-state diffraction model for array of cells (K.Bane et al., SLA

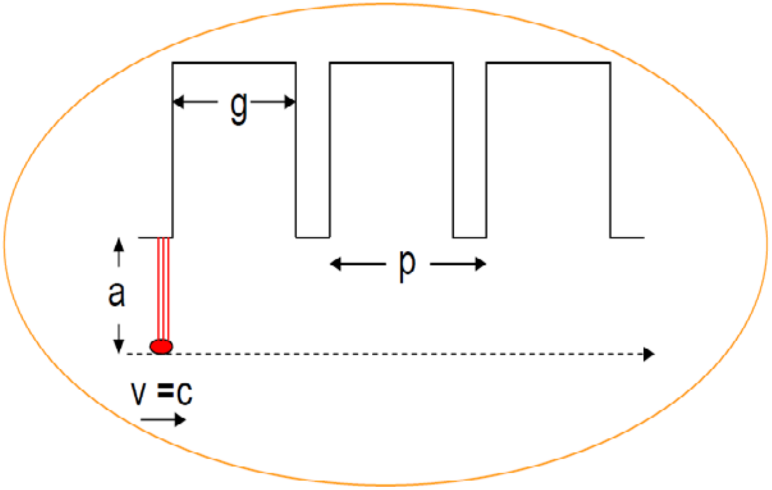
- Infinitely long array of RF cells with cylindrical symmetry.
- The wake is evaluated after traveling a distance ≥ $a^2/2\sigma_z$.
- Good approx. over wide range of cell parameters; very relevant for FEL linacs.

$Z_0 = 120\pi\ \Omega$
(vacuum impedance)

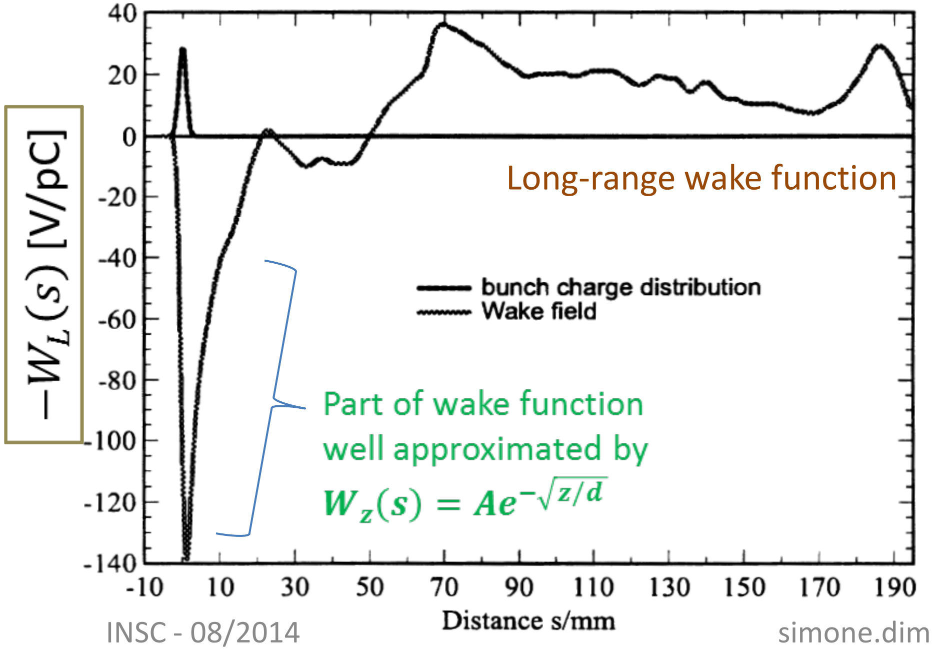
$d = \frac{0.41 a^{1.8} g^{1.6}}{p^{2.4}}$
(geometric factor)

$$w_L(\Delta z) \simeq \frac{Z_0 c}{\pi a^2} e^{-\sqrt{\Delta z/d}}$$

MKS units:
$$\frac{V}{A} \times \frac{m}{s} \times \frac{1}{m^2} = \frac{V}{C \cdot m}$$



**Wakefield through 8-cavity TESLA cryomodule**



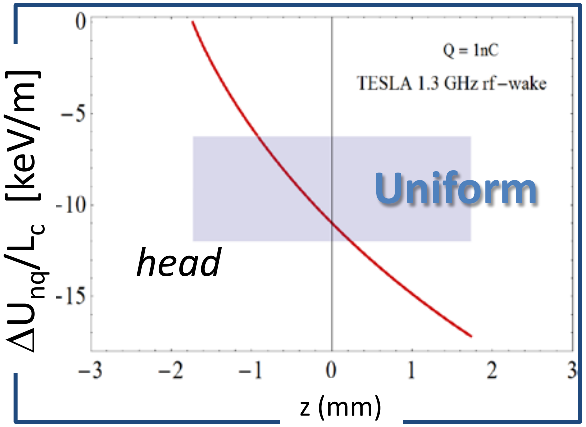
Long-range wake function

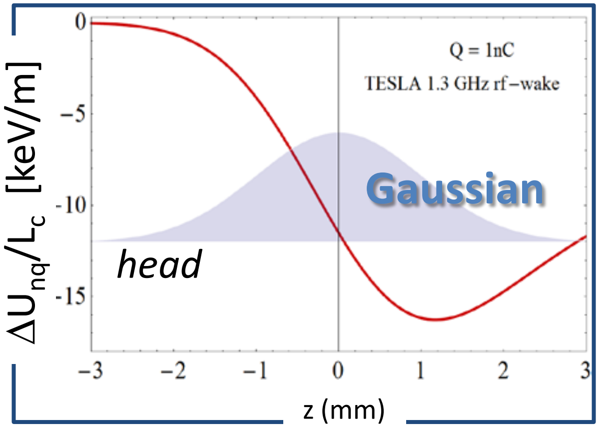
Part of wake function well approximated by $W_z(s) = Ae^{-\sqrt{z/d}}$

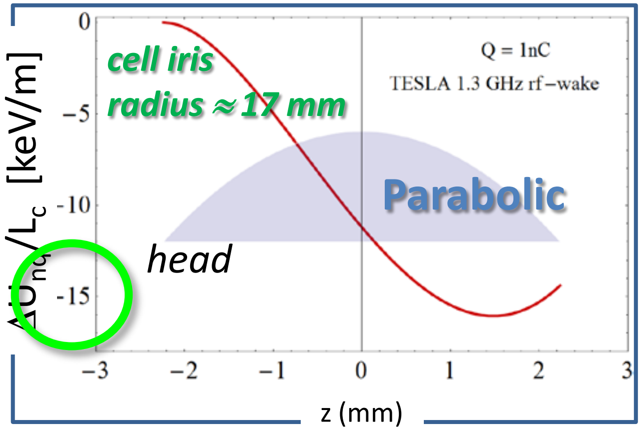
- Accurate determination of wake potential for actual structure design can be done with specialized numerical codes (e.g. MAFIA).
- Fit numerical result with analytical model (e.g., T. *Weiland at al., TESLA Report 2003-19 for the TESLA cav.)*

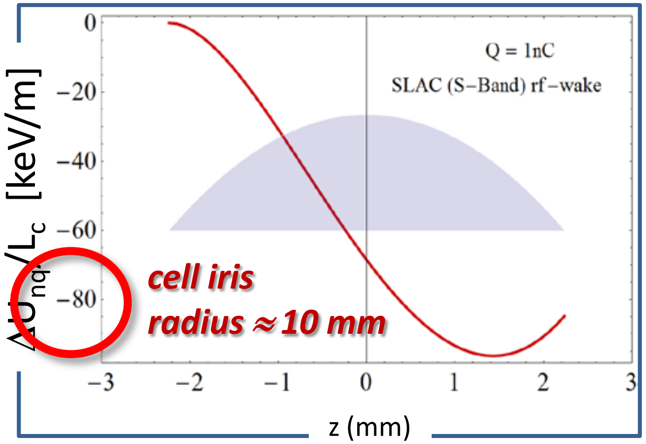
# Dependence on bunch shape (longitudinal) and cell iris

Figs. by M. Venturini







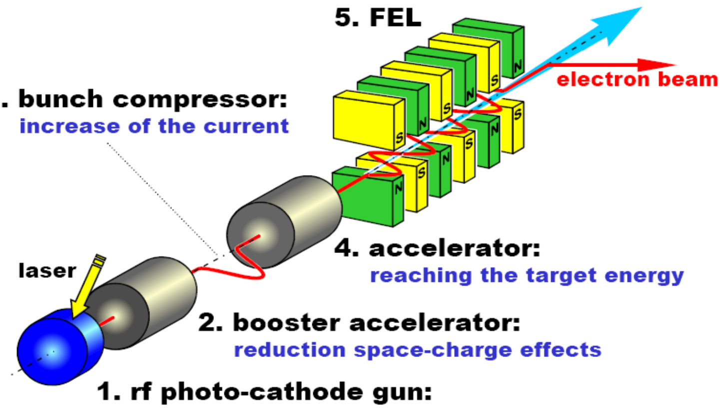


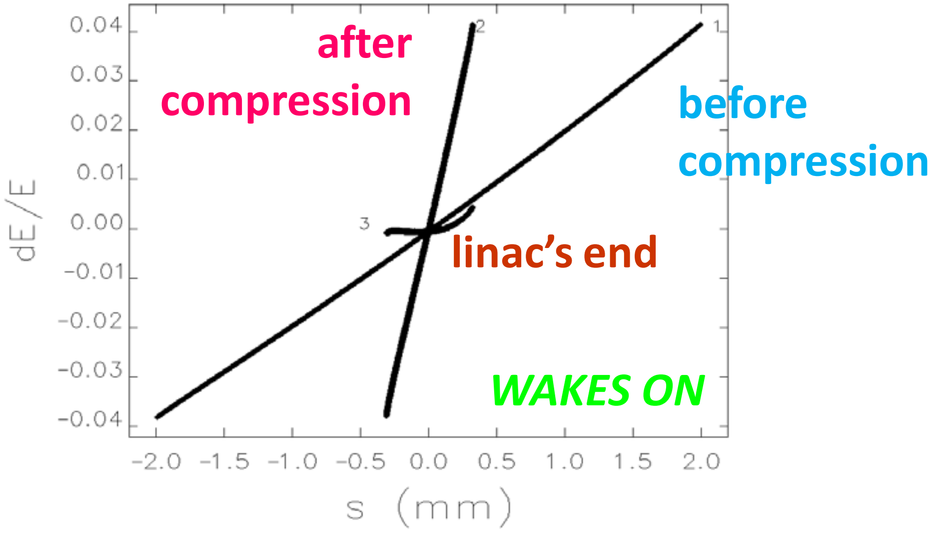
Energy loss along bunch:

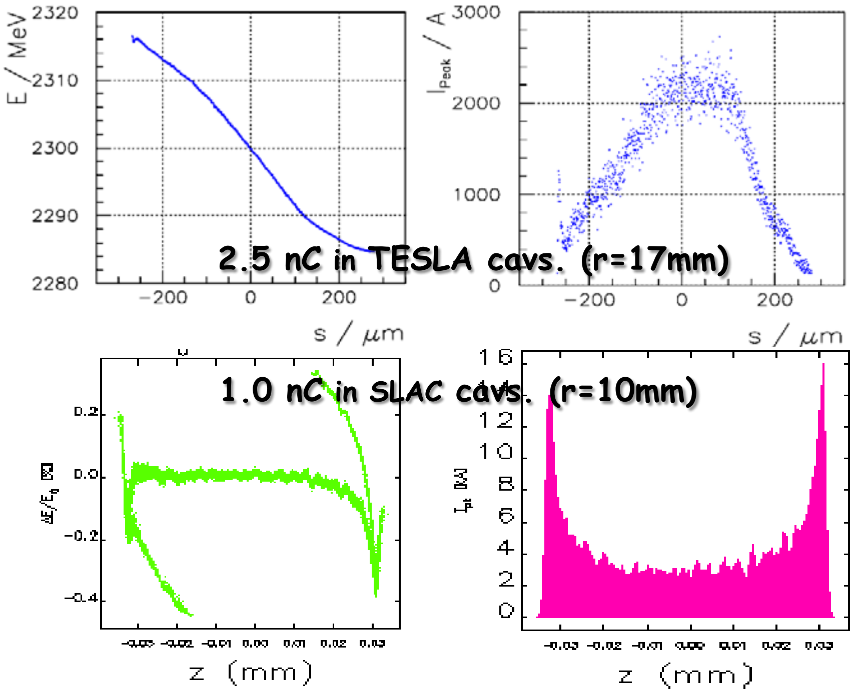
$$\Delta U_{Nq}(z) = U_0 + U_0' z + \frac{U_0'' z^2}{2} + \frac{U_0''' z^3}{6} + \cdots$$

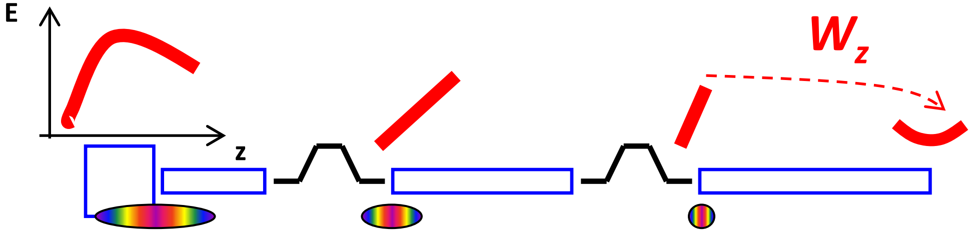
- *$0^{th}$- order:* average energy loss by bunch. Usually not an important issue. Can be compensated by adjusting RF structure voltage.
- *$1^{st}$- order:* affects the linear energy chirp of bunch. Can be compensated by adjusting the RF structure phases.
- *$2^{nd}$ order:* affects the quadratic energy chirp.
- *$3^{rd}$ order* and higher: usually stronger than cubic (and higher order) nonlinearities from RF structure waveform. Contributes to shaping current profile during compression (e.g., current spikes for large compression factors).

# Effects of longitudinal wakefields



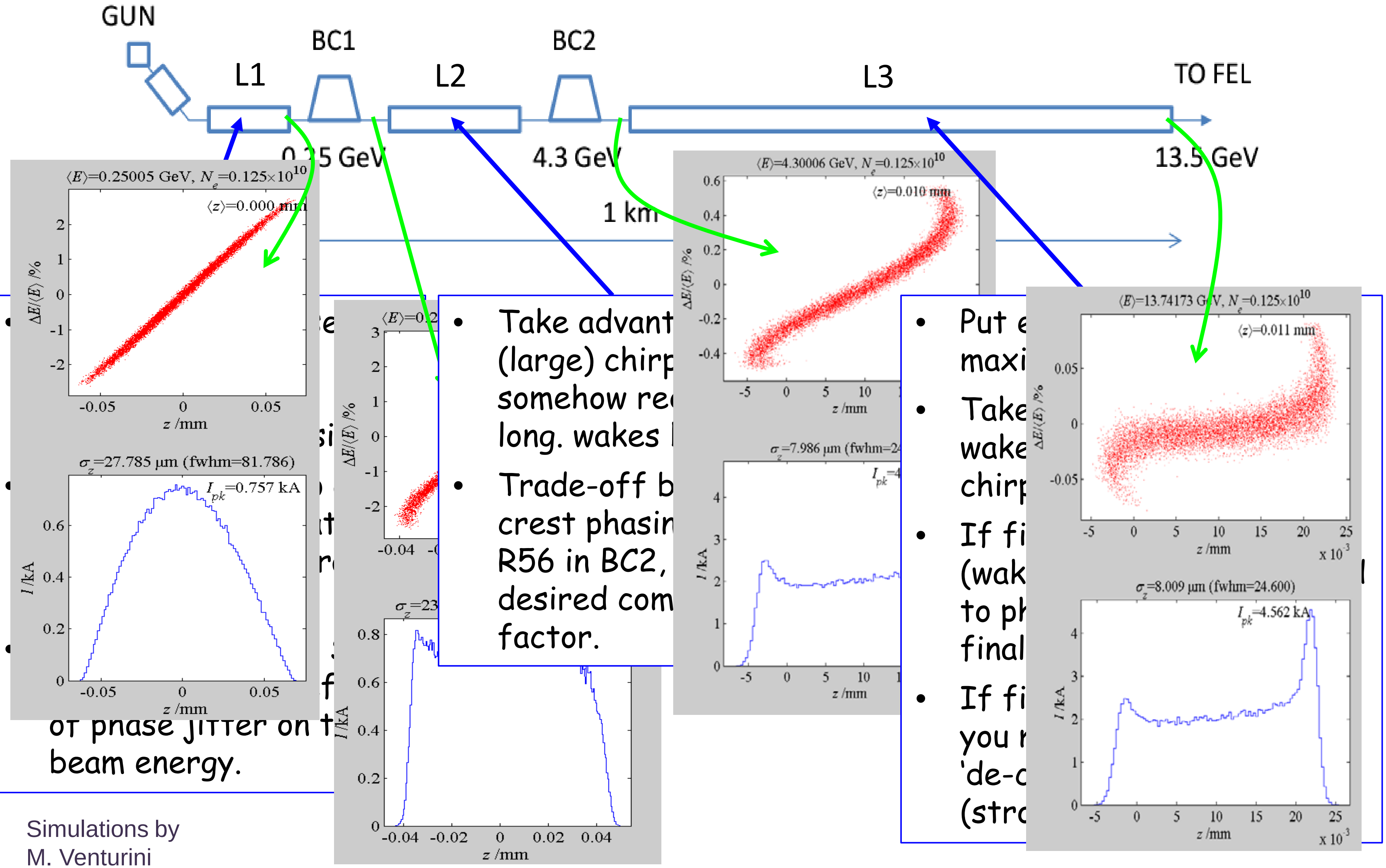






- Smaller iris ~ higher gradients ~ stronger wakefields;
- Stronger wakefields ~ remove the linear chirp (U'), excite edge current spikes (U''' + BC).
- X-band linearizes at 2nd order (cancels U'').

# Optimization strategies in the presence of $w_L$



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Simulations by
M. Venturini

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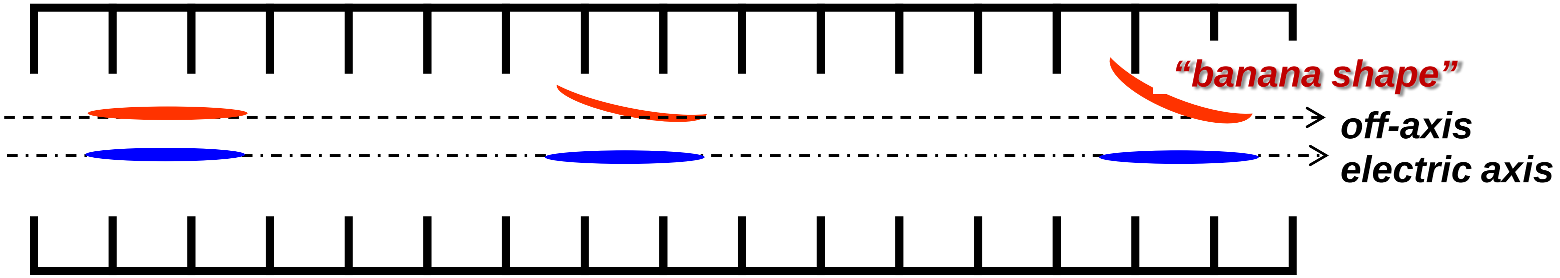
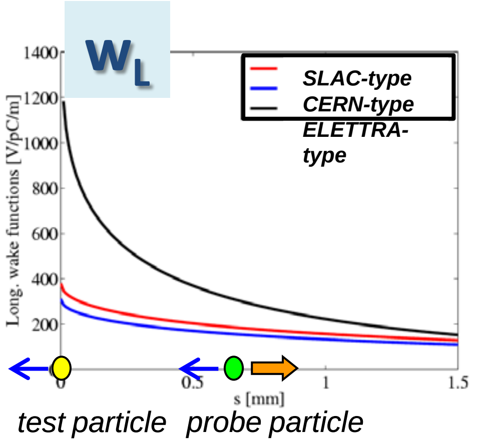
# Transvers wake function model (K.Bane et al., SLAC)

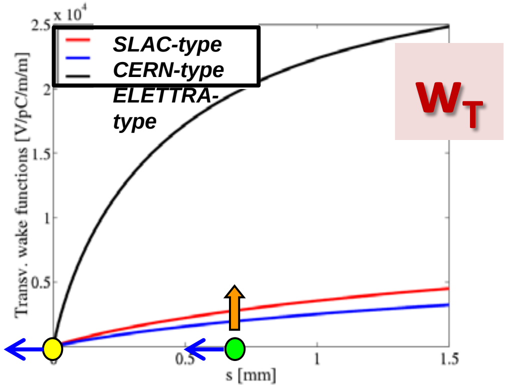
Transverse wakefield is generated as the radial symmetry of the e.m. field brought by the beam is broken ("dipole mode"). Namely, it is excited by a **relative misalignment of the beam respect to the cavity electric axis** (*coherent betatron oscillations*).



$$w_T(z)=\frac{Z_0 c s_1}{\pi a^4}\left[1-\left(1+\sqrt{\frac{z}{s_1}}\right)e^{-\sqrt{\frac{z}{s_1}}}\right]\left[\frac{V}{C\,m^2}\right],$$

- $s_1 \approx 0.3\div0.8$mm is a cell geometric parameter.
- $w_T$ is the wakefield per unit length of the cavity, per unit length of (relative) lateral displacement.





- Note: **$w_L$** is **stronger** for **shorter bunches**, **$w_T$** is stronger for **longer ones**.

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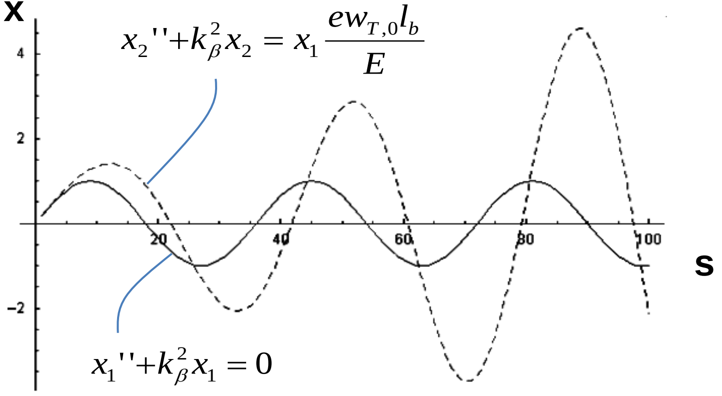
# Single-bunch beam break-up

- Equation of motion for x(z,s) in the presence of $w_T$:

$$\underset{\textbf{acceleration}}{\frac{d}{ds}\left[\gamma(s)\frac{d}{ds}x(z,s)\right]}+\underset{\boldsymbol{\beta}\textbf{-focusing}}{k_\beta^2\gamma(s)x(z,s)}=r_e\int_z^\infty dz'\,\underbrace{\rho(z')}_{\text{charge distribution}}\underbrace{w_T(z'-z)}_{\text{wake function}}\underbrace{\left[x(z',s)-d_c(s)\right]}_{\text{cavity displacement relative to the particle free }\beta\text{-oscillation}}$$

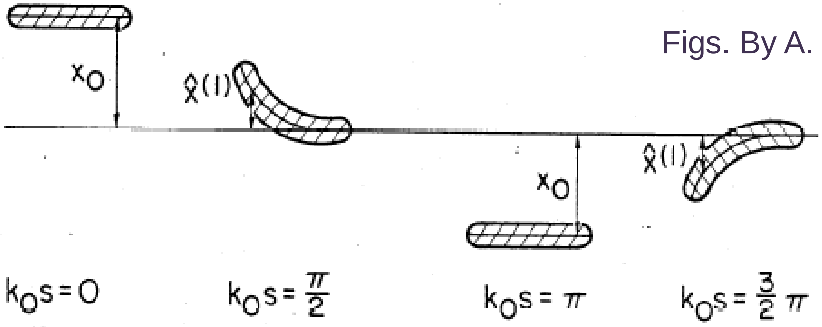
- In **two-particle model**, the bunch head drives resonantly the tail:

*TAIL behaves as a resonantly driven oscillator*



*HEAD obeys Hill's equation*

- **Bunch** longitudinal **slices** feel different wake kicks, which **displace** them **in the transverse phase space**, one respect to the other ⇒ ***projected emittance growth***

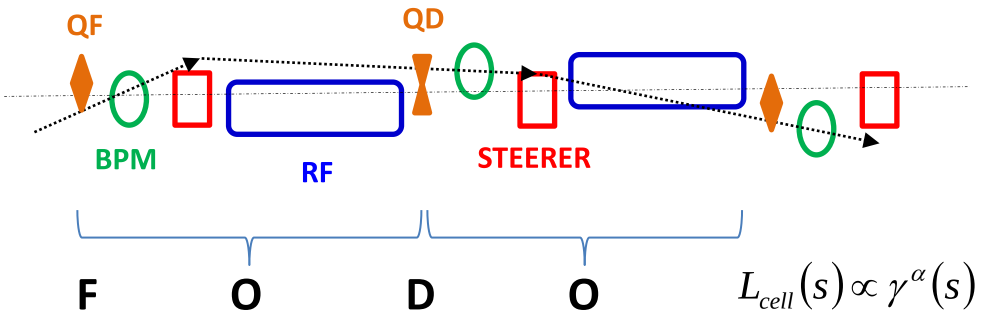


Figs. By A. Chao

$$\varepsilon_r=\frac{4\pi\varepsilon_0}{I_A}\frac{w_{T,0}Il_bL^2}{\gamma_0}$$

From r.h.s. of the e.o.m. we extract a coefficient that measures the **coupling strength** of the wake to the bunch.

# Sources of beam-to-linac misalignment



$L_{cell}(s) \propto \gamma^{\alpha}(s)$

Beam is kicked by misaligned quadrupoles, RF structures and steerers. Centering it into BPMs is usually not the best way to minimize transv. wakes.

Only **BPMs** + **QUADS** randomly misaligned:

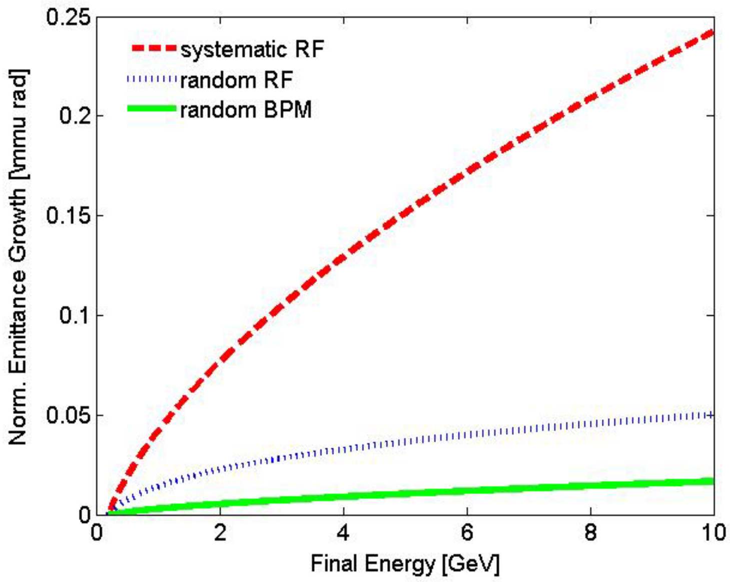
$$\Delta(\gamma\varepsilon) \approx \sigma^2_{y,BPM}\left[\pi\varepsilon_0 r_e N W_\perp(2\sigma_z)\right]^2 \frac{L^2_{cell}}{16\alpha(\Delta\gamma_{str}/L_{str})}\left[\left(\frac{\gamma_f}{\gamma_i}\right)^{2\alpha}-1\right]\frac{\cos(\Delta\mu_{cell}/2)}{\sin^3(\Delta\mu_{cell}/2)}$$

Only **RF structures** randomly misaligned:

$$\Delta(\gamma\varepsilon) \approx \sigma^2_{str}\left[\pi\varepsilon_0 r_e N W_\perp(2\sigma_z)\right]^2 \frac{L_{str}\bar{\beta}}{2\alpha(\Delta\gamma_{str}/L_{str})}\left[\left(\frac{\gamma_f}{\gamma_i}\right)^{\alpha}-1\right]$$

**Pair RF structures** (random + systematic error):

$$\Delta(\gamma\varepsilon) \approx \sigma^2_{str}\left[\pi\varepsilon_0 r_e N W_\perp(2\sigma_z)\right]^2 \frac{L_{cell}\bar{\beta}}{4\alpha(\Delta\gamma_{str}/L_{str})}\left[\left(\frac{\gamma_f}{\gamma_i}\right)^{2\alpha}-1\right]$$



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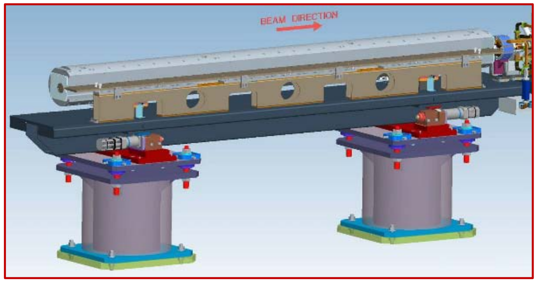
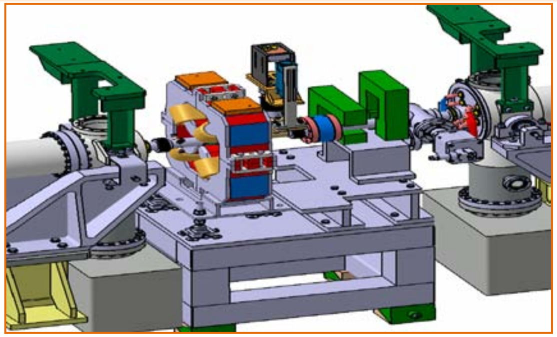
# Strategies for suppressing transverse wakefield instability

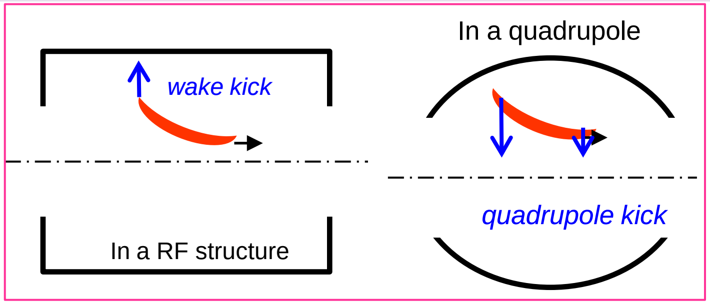
- Accurate and stable **static alignment** of machine components. This involves: fiducialization, piezo, laser tracker; temperature stability, ground motion, etc..



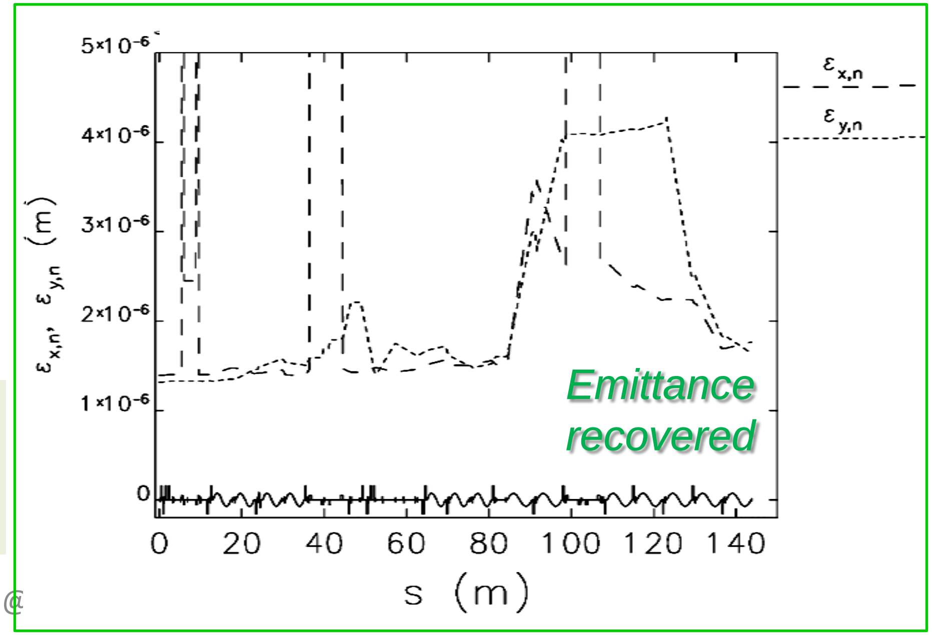
- Define **layout** with BPMs, quads and steerers very close each other and possibly close to the RF structures.



- Induce **energy spread** (e.g., with RF phasing) to **over-focus** the **bunch tail** w.r.t. the head ('Balakin-Novokhatsky-Smirnov damping').



- **Trajectory bumps** to make successive wakefield kicks cancelling each other at the linac end ('emittance bumps').



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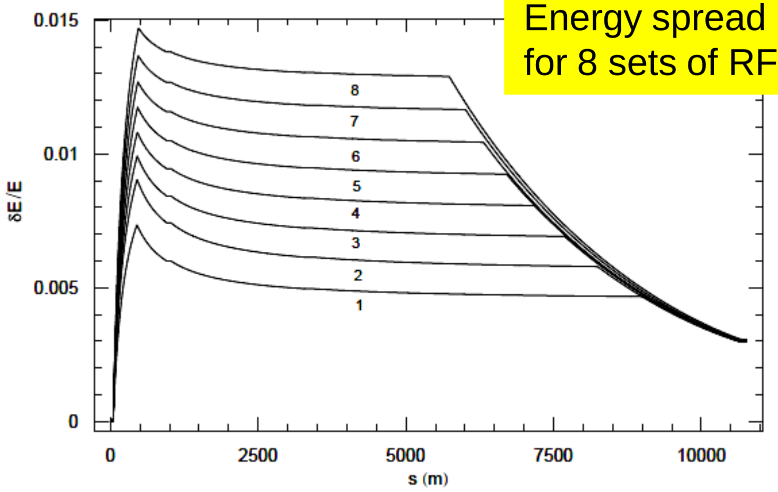
# More on BNS damping

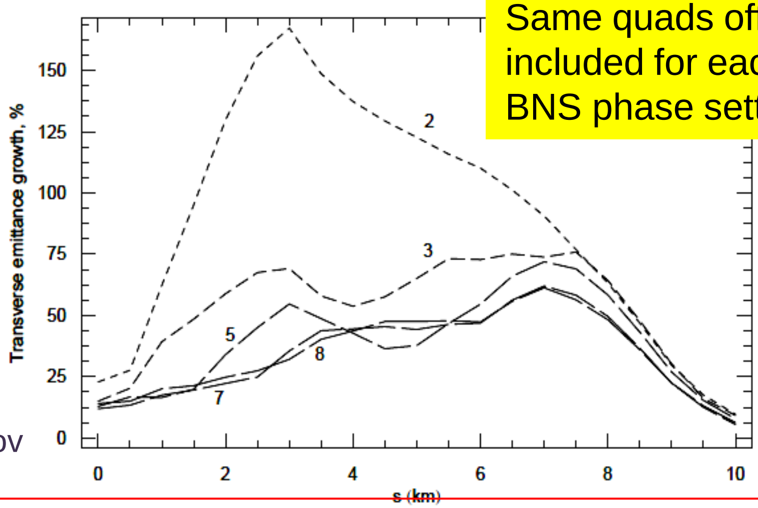
BNS damping requires an energy difference between the head and the tail of the bunch, i.e. a correalted energy spread (chirp). When a FODO lattice is considered, the **auto-phasing RMS energy spread** is:

$$\sigma_{\delta,BNS} \approx \frac{Ne^2 w_T\left(2\sigma_z\right)\bar{\beta}L_{cell}}{E\tan\left(\Delta\mu_{cell}/2\right)}$$

Choose the arrangement of RF phases that minimize the transverse emittance at the linac end:



Energy spread profile for 8 sets of RF phases



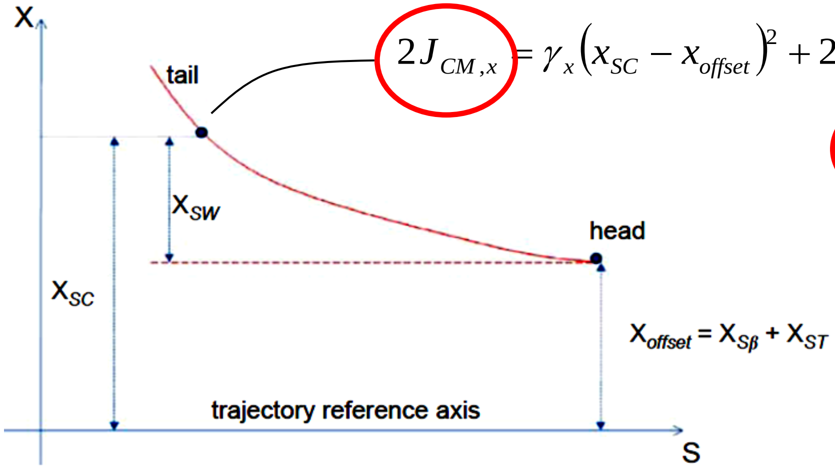
Same quads offset included for each BNS phase setting

Figs. by G. Stupakov

- **Drawback 1:** $\delta_{BNS}$ has opposite sign respect to $\delta$ required for magnetic compression with four-dipole chicanes → additional complexity for the choice of RF phases.
- **Drawback 2:** large $\delta_{BNS}$ along the linac contributes to emittance dilution by spurious dispersion.
- **Drawback 3:** off-crest RF phasing by BNS lowers the final beam energy.

# More on trajectory bumps

- The scheme of bumps for suppressing transv. wakefield may be be affacted by trajectory jitter;
- Tolerance for the trajectory jitter: the position of **each slice centroid varies less than one unperturbed RMS beam size**:



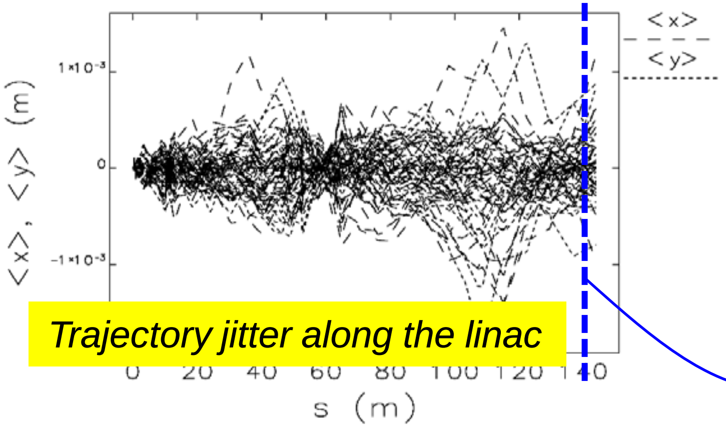
$$2J_{CM,x} = \gamma_x\left(x_{SC} - x_{offset}\right)^2 + 2\alpha_x\left(x_{SC} - x_{offset}\right)\left(x'_{SC} - x'_{offset}\right) + \beta_x\left(x'_{SC} - x'_{offset}\right)^2 \equiv \varepsilon_{SW,x}$$

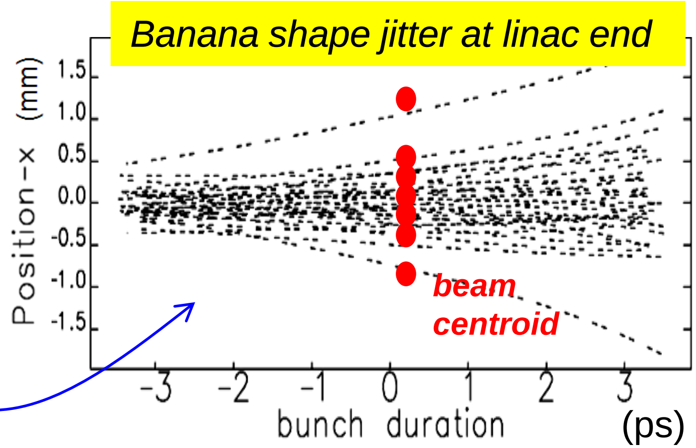
$\frac{\sigma_{x,SC}}{\sqrt{\varepsilon_x \beta_x}} \le 1$, where:

$$\sigma_{x,SC} = \sqrt{\langle x_{SC}^2 - \bar{x}_{SC}^2 \rangle} \simeq \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(x_{SW}^i - \bar{x}_{SW}\right)^2} =$$

$$= \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\sqrt{\beta_x^i \epsilon_{SW,x}^i}\cos\phi_x - \frac{1}{N}\sum_{i=1}^{N}\sqrt{\beta_x^i \epsilon_{SW,x}^i}\cos\phi_x\right]^2} =$$

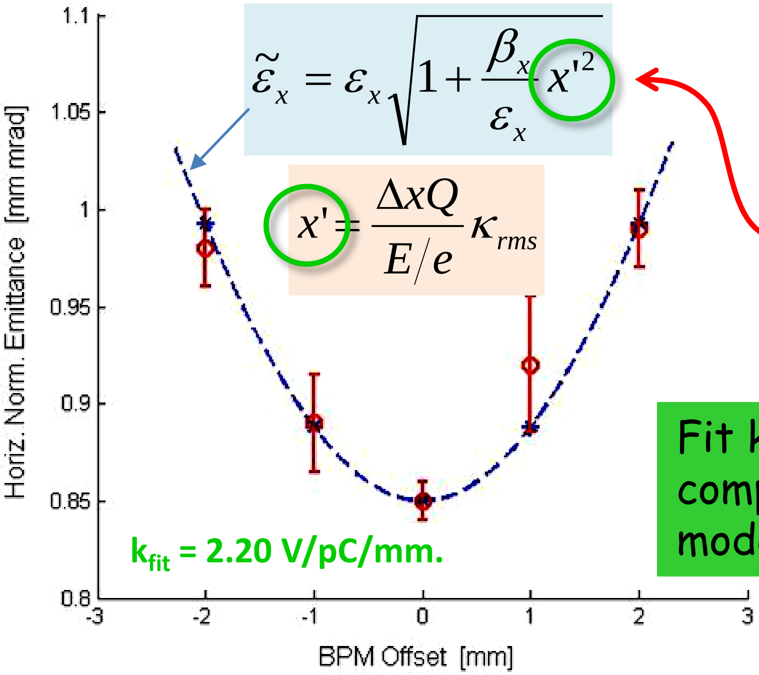
$$= \sqrt{\beta_x}\cos\phi_x\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\sqrt{\epsilon_{SW,x}^i} - \frac{1}{N}\sum_{i=1}^{N}\sqrt{\epsilon_{SW,x}^i}\right]^2}$$


*Trajectory jitter along the linac*


*Banana shape jitter at linac end*

# Geometric 'transverse kick factor' in a collimator

**Collimators** are metallic blocks with **small apertures** to intercept, scatter and absorb undesired particles at very large β-amplitudes ("halo" particles, $|A| \geq 20\sigma$) or off-energy ($|\delta| \geq 2\%$). Doing so, they **protect** the **undulator** from being hitted by e.m. showers generate by primary (halo) or secondary particles (from vacuum chamber). The *beam core should* pass through the collimators *untouched*.



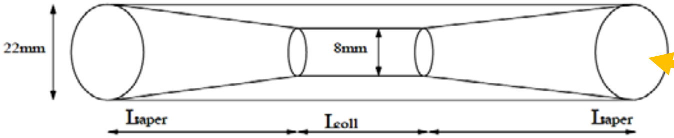
$$\tilde{\varepsilon}_x = \varepsilon_x \sqrt{1 + \frac{\beta_x}{\varepsilon_x} x'^2}$$

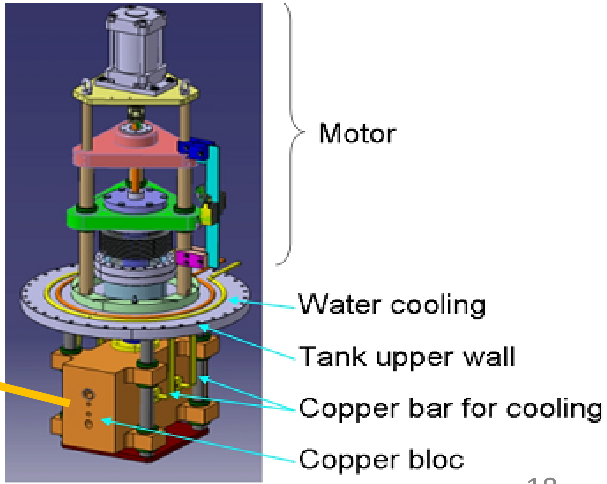
$$x' = \frac{\Delta x Q}{E/e} \kappa_{rms}$$

$k_{fit}$ = 2.20 V/pC/mm.

- **How to measure $w_T$ in a collimator:**
  - i. vary beam offset inside it and record the trajectory variation downstream (***average*** $w_T$-kick along the bunch);
  - ii. vary the beam offset inside it and measure the emittance downstream (***rms*** $w_T$-kick along the bunch).

Fit $k_{rms}$ and compare it with models available.





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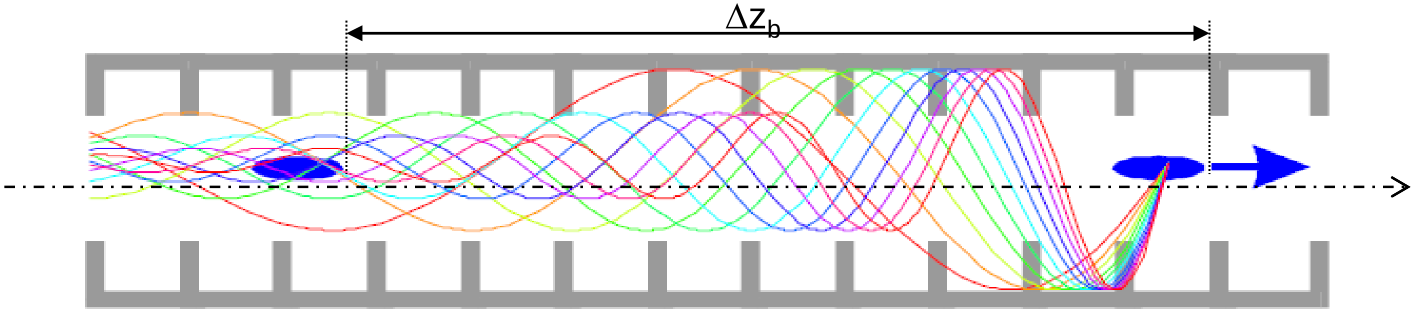
# Long-range wakefield and multi-bunch beam break-up

- The long-range (transverse) wakefield is the extension of the short-range to multi-bunch patterns. Now, leading and trailing *particles* in the *same bunch* are substitued with leading and trailing ***bunches*** in the same ***bunch train***.



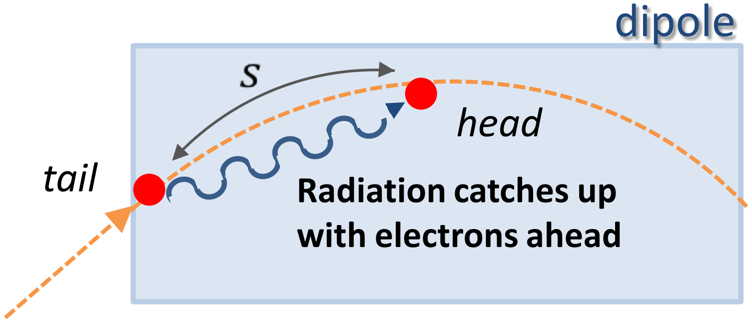
- Tend to consider the wakefield as the sum of many ***field modes*** (HOMs) excited by the first bunches of a train which travel off axis in the accelerating structure:

$$w_T(z) = \sum_k \frac{r_{s,k}\omega_k}{Q_k} e^{-\frac{\omega_k z}{2cQ_k}} \sin\frac{\omega_k z}{c} \left[\frac{V}{Cm^2}\right] \quad \Rightarrow \Delta x_f \propto e^{\sqrt{Qw_T}}$$

(instability)

- The HOMs can be damped with **special design** of the **RF structures**:
  - **detuned structures** have slightly different cell-to-cell dimensions to introduce a frequency spread of each mode, causing decoherence of the wake function;
  - very **low Q** (~20) choke mode structures (NC linacs);
  - **HOM loop couplers** (SC linacs) to couple out frequency modes < GHz and absorb this power in room temperature loads.

# Coherent Synchrotron Radiation: longitudinal wakefield



**Radiation catches up with electrons ahead**

Closed-form analytical expression for electric field along direction of motion:

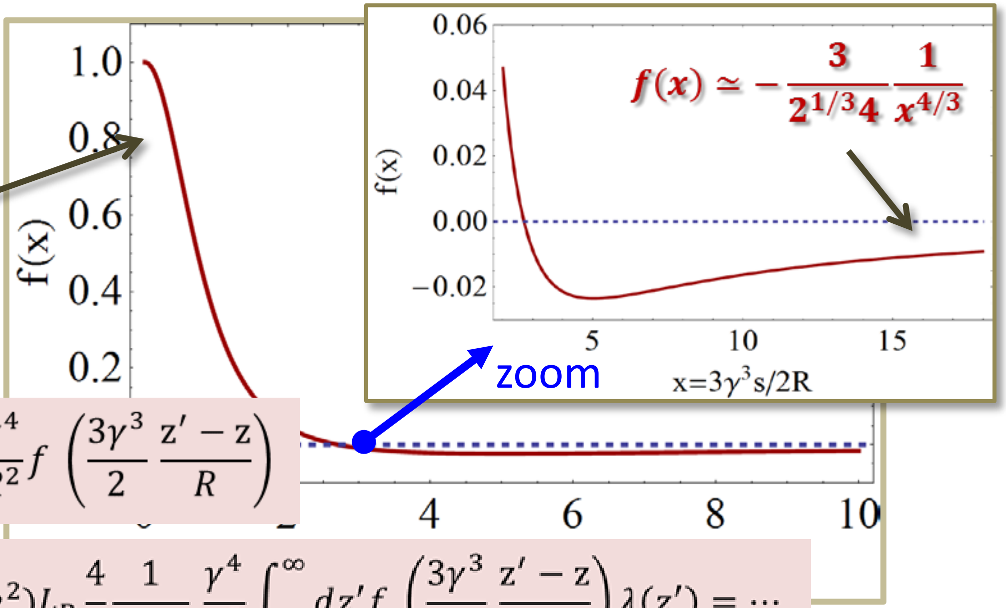
- two particles on same trajectory path,
- uniform circular motion (steady-state),
- use expressions for retarded-fields

$$E_{z,CSR} = -\frac{4}{3}\frac{e}{4\pi\varepsilon_0}\frac{\gamma^4}{R^2} \times f(x)$$

$$f(x) = 0 \text{ for } x < 0$$

$$\int_0^\infty f(x)dx = 0$$

$$f(x) \simeq 1 - \frac{14}{9}x^2$$



**WAKE FUNCTION**

$$W_{L,CSR}(z'-z) = -\frac{1}{q}\int_{s_1}^{s_2} ds\, E_{z,CSR} = -L_B\frac{4}{3}\frac{1}{4\pi\varepsilon_0}\frac{\gamma^4}{R^2}f\left(\frac{3\gamma^3}{2}\frac{z'-z}{R}\right)$$

$$\Delta U(z) = -Ne^2\int_z^\infty dz' W_{L,CSR}(z'-z)\lambda(z') = (Ne^2)L_B\frac{4}{3}\frac{1}{4\pi\varepsilon_0}\frac{\gamma^4}{R^2}\int_z^\infty dz' f\left(\frac{3\gamma^3}{2}\frac{z'-z}{R}\right)\lambda(z') = \cdots$$

**ENERGY LOSS ALONG BUNCH**

$$\ldots \cong -\frac{Ne^2}{4\pi\varepsilon_0}\frac{2}{3^{\frac{1}{3}}}\frac{L_B}{R^{\frac{2}{3}}}\int_z^\infty \frac{dz'}{(z'-z)^{1/3}}\frac{d\lambda(z')}{dz'}$$

for $x \gg 1$

***integrable singularity***

Courtesy of M. Venturini

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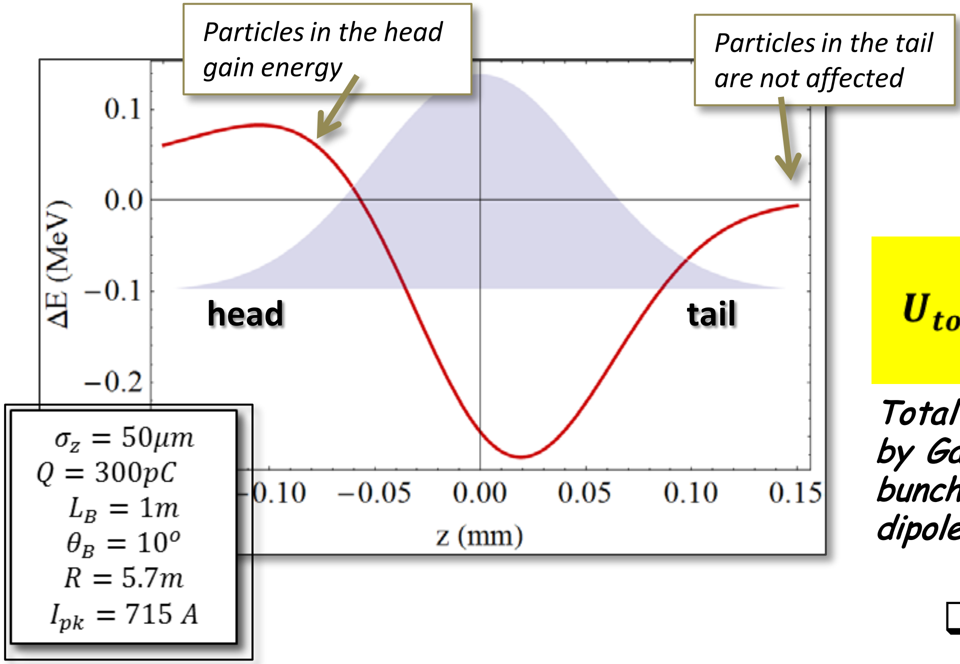
# Gaussian bunch, Transient effects



$\sigma_z = 50\mu m$
$Q = 300pC$
$L_B = 1m$
$\theta_B = 10^o$
$R = 5.7m$
$I_{pk} = 715\ A$

Proportional to 1/3 power of bend radius

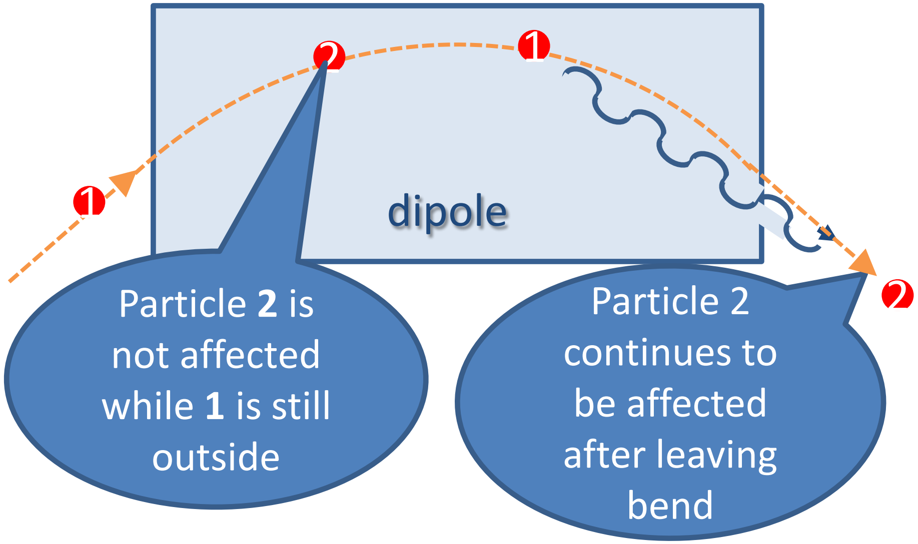
Proportional to bend angle

$$U_{tot} = -0.028 \times Ne^2 Z_0 c \times \frac{R^{1/3}\theta_B}{\sigma_z^{4/3}} = -0.16\text{MeV}$$

*Total energy loss by Gaussian bunch through dipole*

Inversely proportional to 4/3 power bunch length

- ☐ Steady-state model doesn't account for transient effects (in and out of dipoles):
- there are 1D models accounting for them and implemented in tracking codes.
- In practice, the RF settings will have to compensate for linear and nonlinear effects induced by CSR.
- The most notable effect of the longitudinal CSR wake is on the transverse dynamics.



Courtesy of M. Venturini

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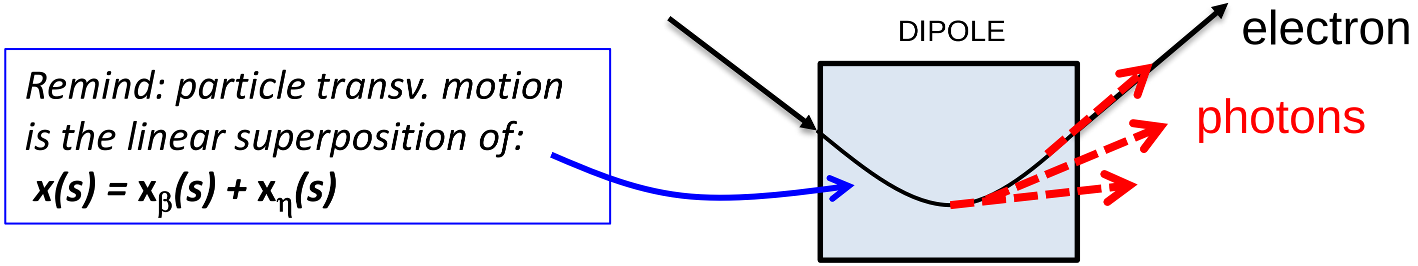
# CSR effect on particle trajectory

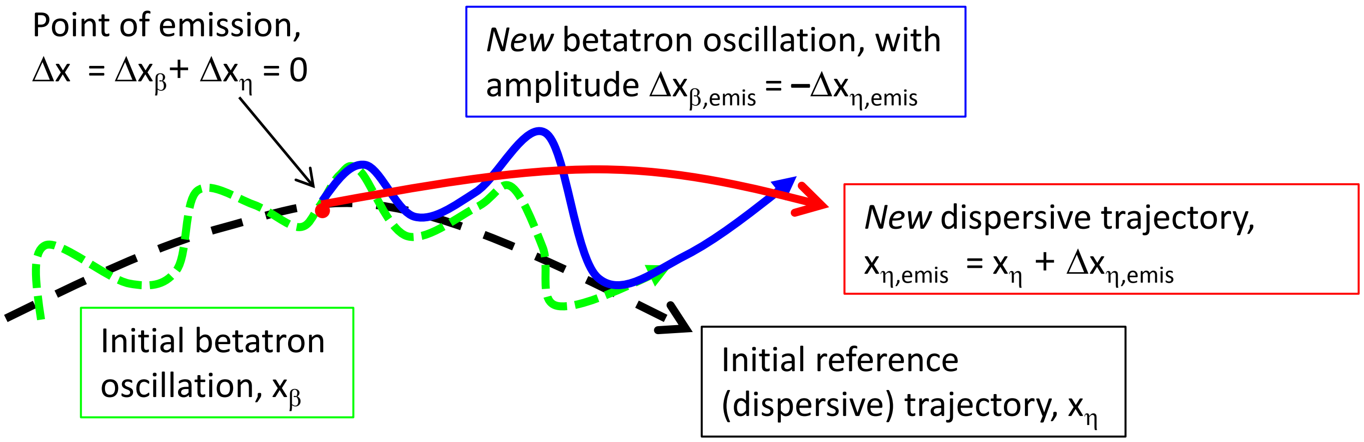
In the following, we will adopt a simplified picture for the CSR transverse effect. Experimental results suggest that it is accurate enough for describing *most* of the practical cases.

**1.** *Photons* are *emitted* in the *beam direction of motion*, at any point along the curved trajectory in a dipole magnet ⇒ **CSR longitudinal 'kick'**, $p_z(s) \rightarrow p_z(s) - \delta p_{z,CSR}(s)$.



*Remind: particle transv. motion is the linear superposition of:*
***$x(s) = x_\beta(s) + x_\eta(s)$***

**2.** At any point of emission, particle ***transverse position does not change***: **$\Delta x = 0$**. Since the emission happens in an energy-dispersive region, $\Delta x_\beta = -\Delta x_\eta$. That is, the particle **starts β-oscillating** ($\Delta x_\beta$) **around a *new* dispersive trajectory** ($-\Delta x_\eta$).



Point of emission,
$\Delta x = \Delta x_\beta + \Delta x_\eta = 0$

*New* betatron oscillation, with amplitude $\Delta x_{\beta,emis} = -\Delta x_{\eta,emis}$

*New* dispersive trajectory,
$x_{\eta,emis} = x_\eta + \Delta x_{\eta,emis}$

Initial betatron oscillation, $x_\beta$

Initial reference (dispersive) trajectory, $x_\eta$

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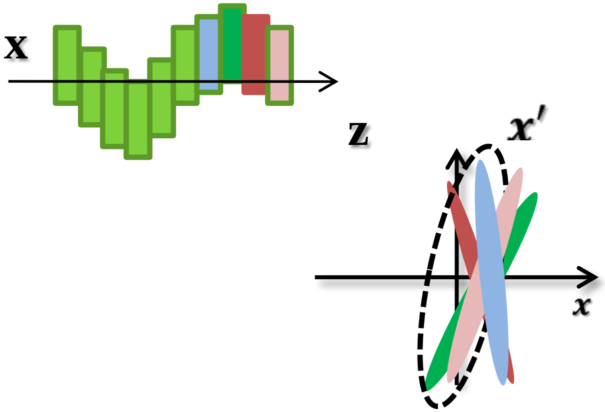
# Emittance growth in the single-kick approximation



**3.** $\delta p_{z,CSR}$ is **correlated with *z*** along the bunch, as depicted by $\mathbf{w}_{L,CSR}$ (see previous slides):

- **different slices feel different CSR kicks,**
  ⇒ ***the slice centroid's invariant is increased (projected emittance)***
- **all particles at the same z (slice) feel the same CSR kick,**
  ⇒ ***the slice emittance is preserved*** (see later)

**4.** The **projected emittance** is increased by the slices misalignment in the phase space (bending plane only). Use the 'beam matrix' to compute the CSR single-kick effect:

$$\varepsilon \cong \left[\det\begin{pmatrix} \overbrace{\varepsilon_0\beta}^{\langle x_\beta^2\rangle}+\overbrace{\eta^2\sigma^2_{\delta,CSR}}^{\langle \Delta x_\beta^2\rangle=\langle \Delta x_\eta^2\rangle} & -\varepsilon_0\alpha+\eta\eta'\sigma^2_{\delta,CSR} \\ -\varepsilon_0\alpha+\eta\eta'\sigma^2_{\delta,CSR} & \varepsilon_0\dfrac{1+\alpha^2}{\beta}+\eta'^2\sigma^2_{\delta,CSR} \end{pmatrix}\right]^{1/2} = \varepsilon_0\sqrt{1+\frac{H}{\varepsilon_0}\sigma^2_{\delta,CSR}}\,,$$

$H=\left[\eta^2+(\beta\eta'+\alpha\eta)^2\right]/\beta$ takes care of the coupled betatron and dispersive motion;

$\sigma^2_{\delta,CSR}=\left\langle\left(\frac{\Delta E}{E}\right)^2\right\rangle$ is the CSR-induced energy spread relative to the reference energy and it goes like $\propto 1/\sigma_z^\tau$ where $\tau\geq 1$ ($\tau$=4/3 for Gaussian bunch).

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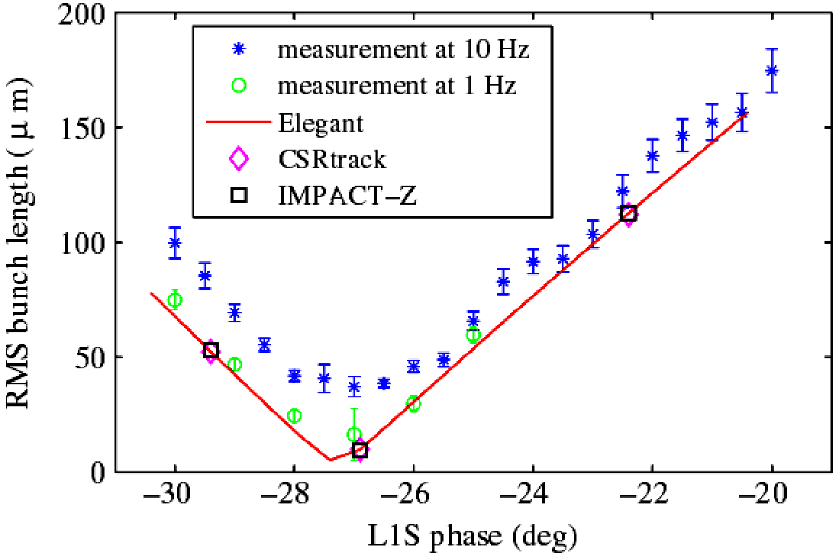
# 4-dipoles magnetic compressor

- Assume the following realistic assumptions:
  - Small bending angle, $\theta \ll 1$;
  - Beam waist in the second half of the chicane.

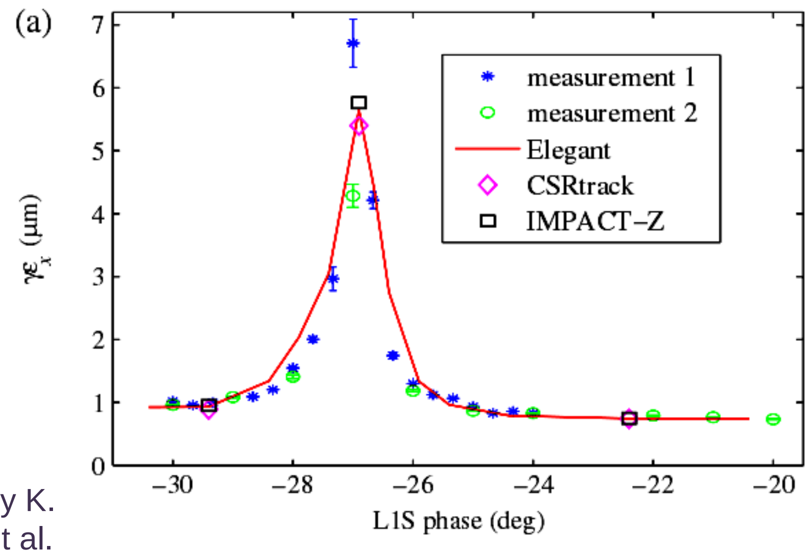
- $\sigma_z$ shorter ($\sigma_{\delta,CSR}$ larger) between 3rd and 4th dipole;
- H-function is larger at the entrance of the 4th dipole, and $H \approx \beta\theta^2$.

**Note:** *CSR* propagation in *drifts* can be important, but it is *neglected here*!

$$\varepsilon = \varepsilon_0\sqrt{1+\frac{H}{\varepsilon_0}\sigma^2_{\delta,CSR}} \approx \varepsilon_0\left(1+\frac{\breve{\beta}\theta^2\sigma^2_{\delta,CSR}}{2\varepsilon_0}\right)_{4th\ dipole}$$



Figs. By K. Bane et al.



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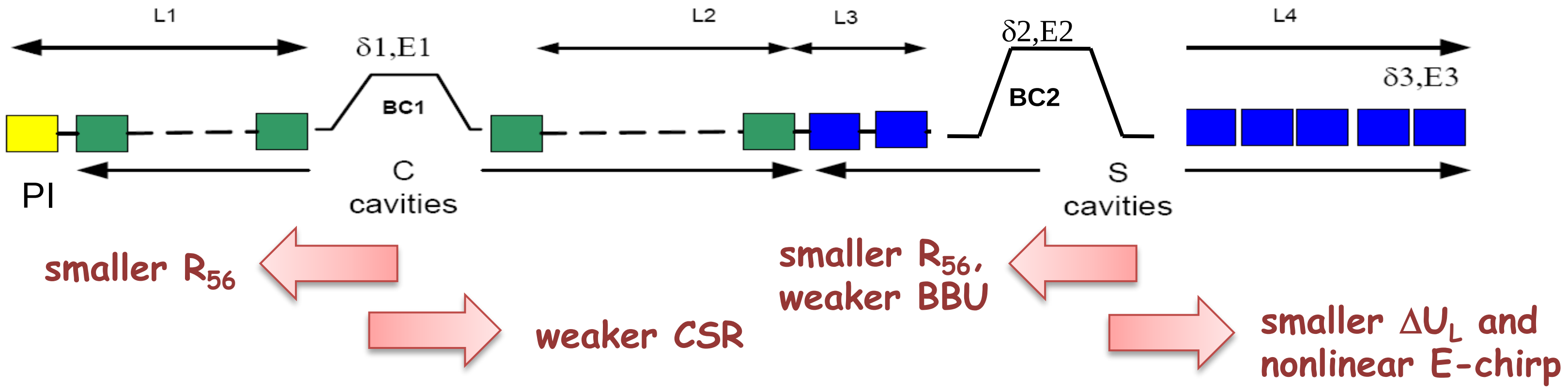
# Optimization of the compressor design

- **At which energy shall I insert my magnetic chicanes?**



smaller $R_{56}$

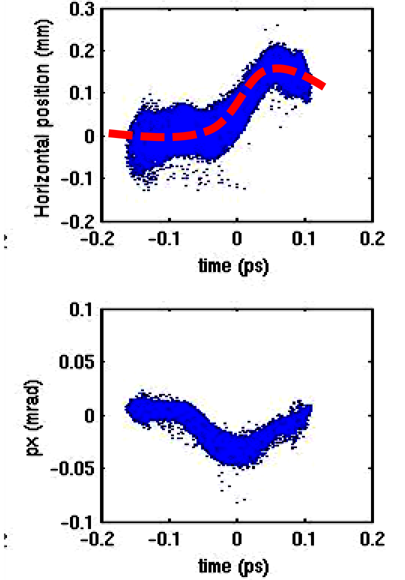
weaker CSR

smaller $R_{56}$, weaker BBU

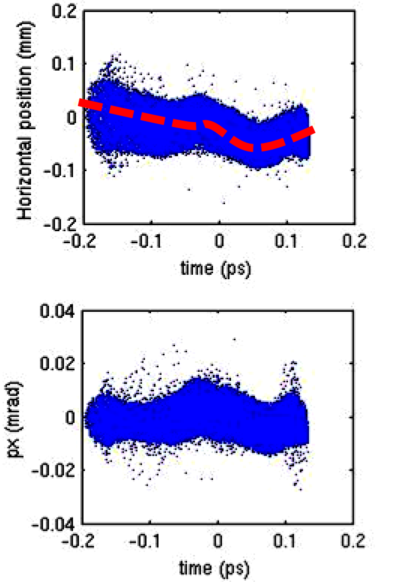
smaller $\Delta U_L$ and nonlinear E-chirp

- **Which geometry for the four dipoles?**

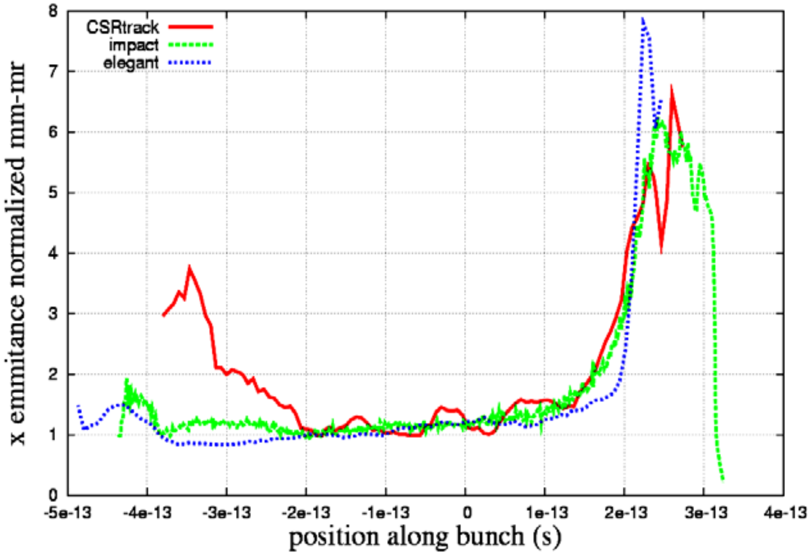
***C-type* chicane**



***S-type* chicane**

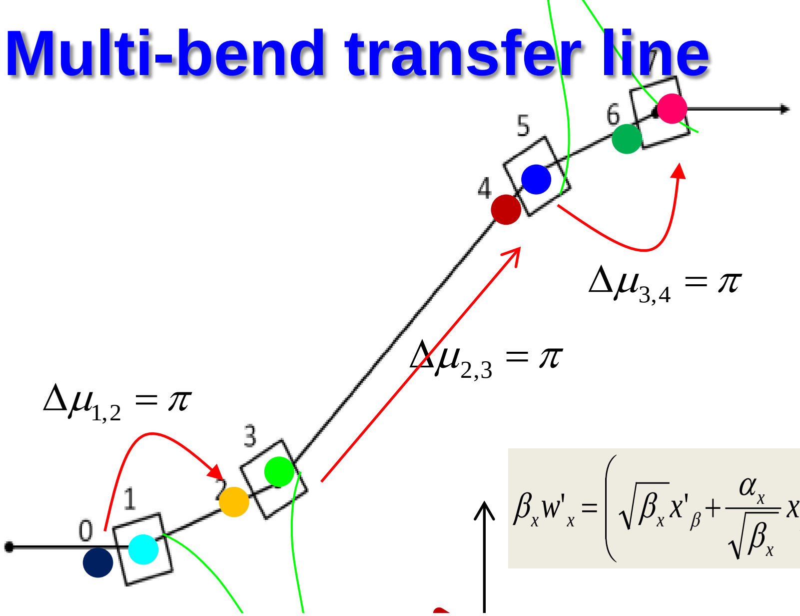


- **Are 3D and transient effects important?**



Figs. by R. Bartolini

# Multi-bend transfer line



$$\beta_x w'_x = \left( \sqrt{\beta_x} x'_\beta + \frac{\alpha_x}{\sqrt{\beta_x}} x_\beta \right)$$

*Double bend achromatic lines are often used to bring the e-beam from the linac end to the undulator. Optics design may include: achromaticity, isochronicity, matching section, time-compression, diagnostics, collimation....*

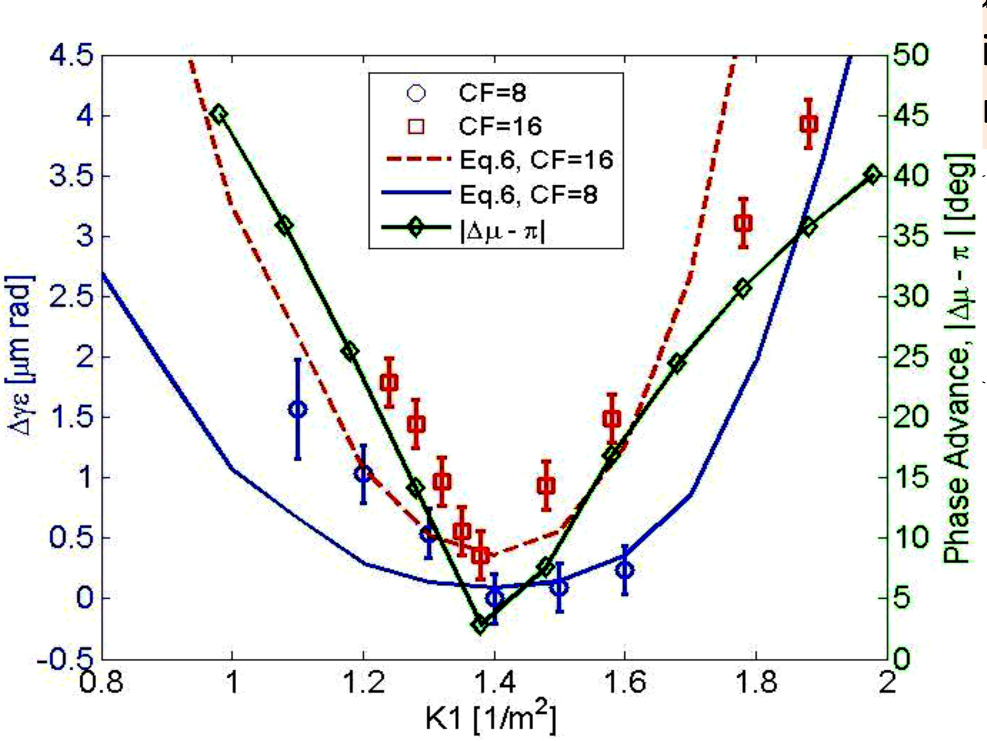
1. Write down the particle coordinates (x,x') with **C-S formalism**. Initial invariant is zero.
2. While traversing a dipole, add the CSR induced η-terms to the initial β-coordinates. is corresponds to an **increase of the** **rticle C-S invariant**:

$$x_1 = \eta\delta \equiv \sqrt{2J_1\beta_1}\cos\Delta\mu\Big|_{\Delta\mu=0} = \sqrt{2J_1\beta_1}$$

$$x_1^{'} = \eta'\delta \equiv -\sqrt{\frac{2J_1}{\beta_1}}\left(\alpha_1\cos\Delta\mu + \sin\Delta\mu\right)\Big|_{\Delta\mu=0} = -\alpha_1\sqrt{\frac{2J_1}{\beta_1}}$$

$$\Downarrow$$

$$2J_1 = \gamma_1 x_1^2 + 2\alpha_1 x_1 x_1^{'} + \beta_1 x_1^{'2} = H_1\delta_{CSR}^2$$



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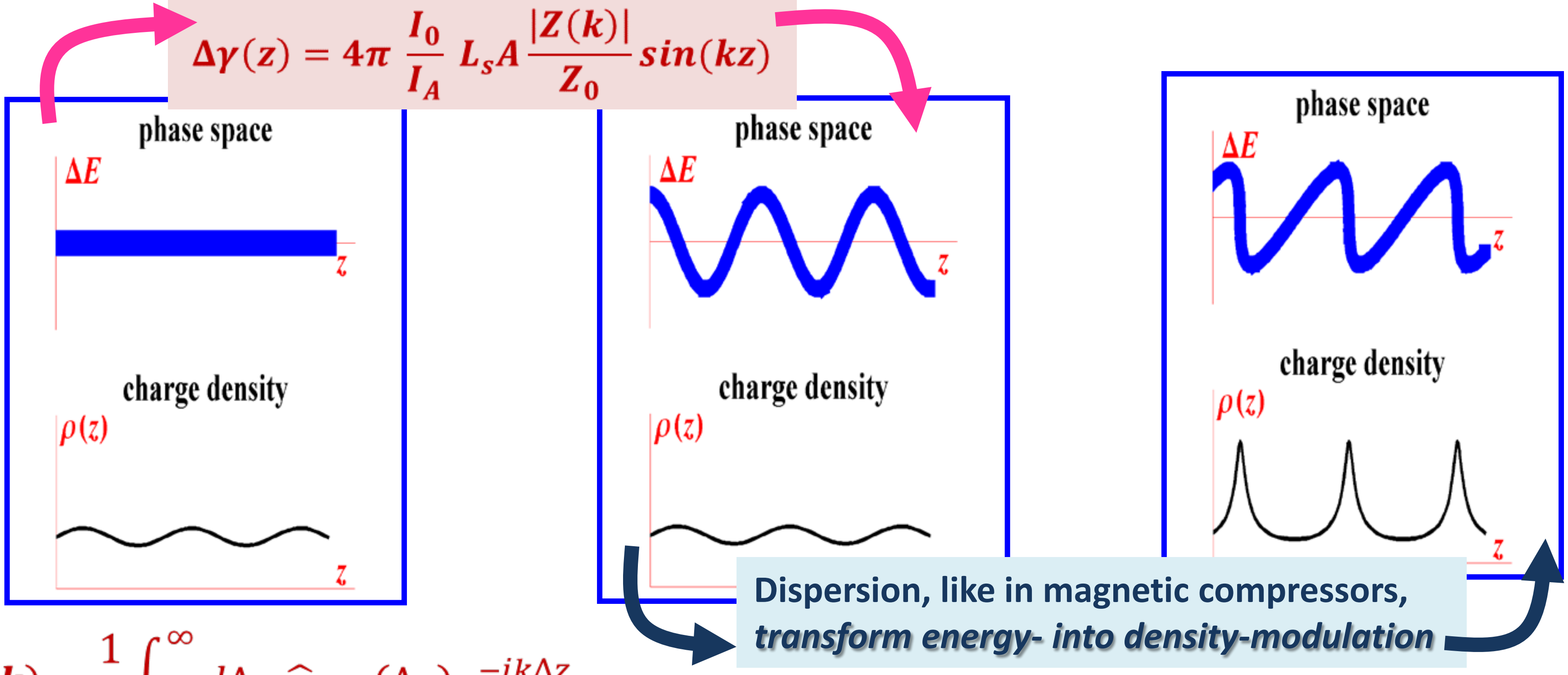
# LSC impedance (frequency domain)

1. Suppose we have a high frequency perturbation with wavenumber $k = 2\pi/\lambda$ on a beam: $I(z) = I_0[1 + A\,cos(kz)]$.
2. Density wave induces energy modulation $\Delta\gamma=\Delta E/mc^2$ over a distance $L_s$ (rigid bunch; ultra-relativistic approx.) ⇒ define the ***LSC impedance per unit length:***

$$\Delta\gamma(z) = 4\pi \frac{I_0}{I_A} L_s A \frac{|Z(k)|}{Z_0} sin(kz)$$



Dispersion, like in magnetic compressors, ***transform energy- into density-modulation***

$$\hat{Z}(k) = \frac{1}{c}\int_{-\infty}^{\infty} d\Delta z\, \hat{w}_{LSC}(\Delta z)e^{-ik\Delta z} =$$

$$= \frac{iZ_0}{\pi\gamma r_b}\frac{1-\xi_b K_1(\xi_b)}{\xi_b},$$

with: $\hat{w}_{LSC}(\Delta z) \equiv \frac{w_{LSC}(\Delta z)}{L}$,
$\xi_b = kr_b/\gamma$, and $K_1$ modified Bessel function

"Microbunching Instability", see later

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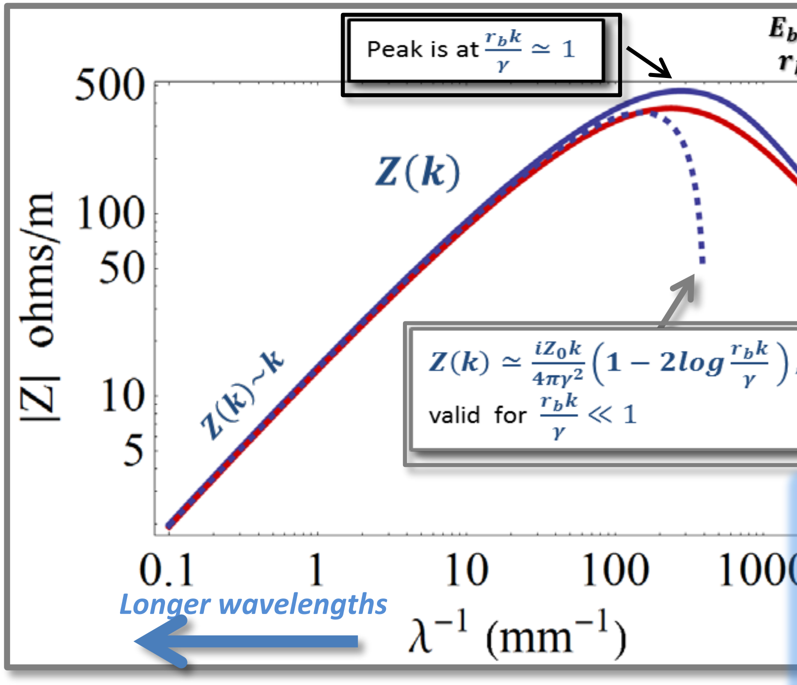
# Collective effects: impedance budget

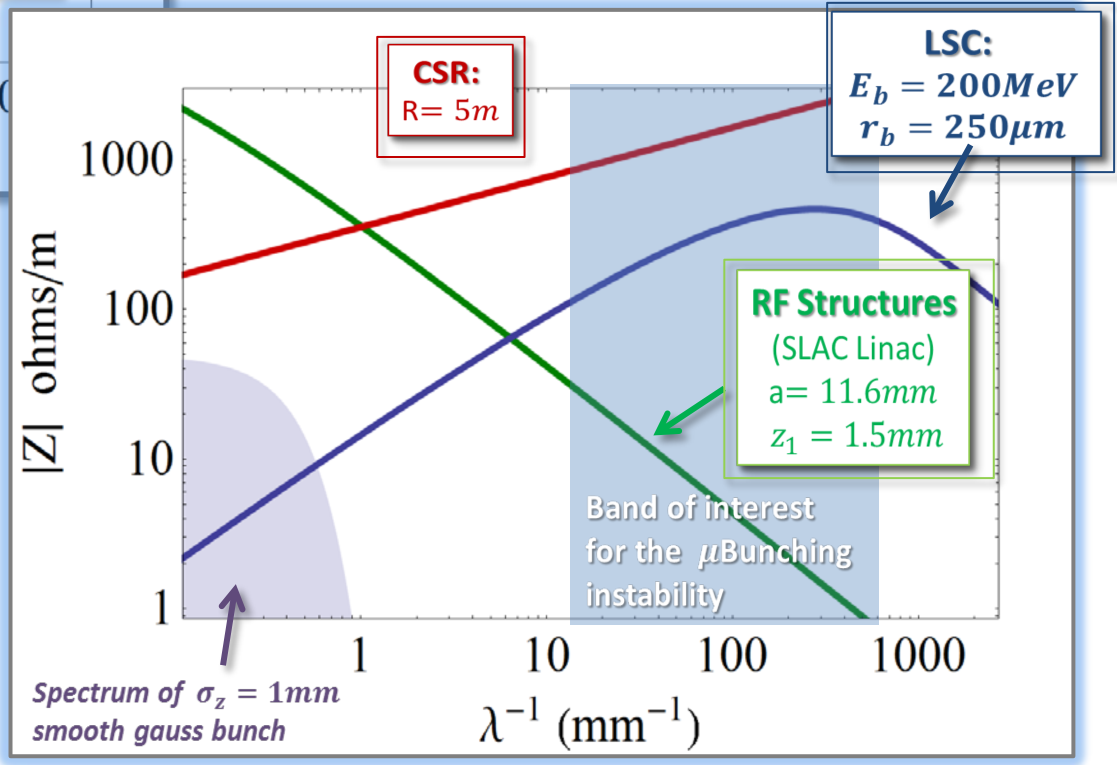

Peak is at $\frac{r_b k}{\gamma} \simeq 1$

$E_b = 200MeV$
$r_b = 250\mu m$

$Z(k) \simeq \frac{iZ_0 k}{4\pi\gamma^2}\left(1 - 2log\frac{r_b k}{\gamma}\right),$
valid for $\frac{r_b k}{\gamma} \ll 1$

Courtesy of
M. Venturini

- CSR impedance is the largest at high frequencies but overall CSR effect is smaller than LSC because dipoles are short compared to rest of machine.
- RF impedance dominates at low frequencies (macroscopic effects on the bunch).


**CSR:**
R= $5m$

**LSC:**
$E_b = 200MeV$
$r_b = 250\mu m$

**RF Structures**
(SLAC Linac)
a= $11.6mm$
$z_1 = 1.5mm$

Band of interest for the $\mu$Bunching instability

Spectrum of $\sigma_z = 1mm$ smooth gauss bunch

$$Z_{CSR} = \frac{Z_0}{\pi R}(0.41 + i0.23)(kR)^{1/3}$$

$$Z_{LSC} = \frac{iZ_0}{\pi\gamma r_b}\frac{1-\xi_b K_1(\xi_b)}{\xi_b}$$

$Z_{RF}$ associated with:

$$w_z = \frac{Z_0 c}{\pi a^2}\exp\left(-\sqrt{z/z_1}\right)$$

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# Microbunching instability

- Small irregularities of charge density (e.g., shot noise) are the most fundamental source of the instability.
- First observed in simulations (M. Borland); Importance pointed out by Saldin et al.. Early 2000s.
- Seeded by irregularities in longitudinal beam densities, amplified primarily by LSC + presence of dispersive sections (BCs).

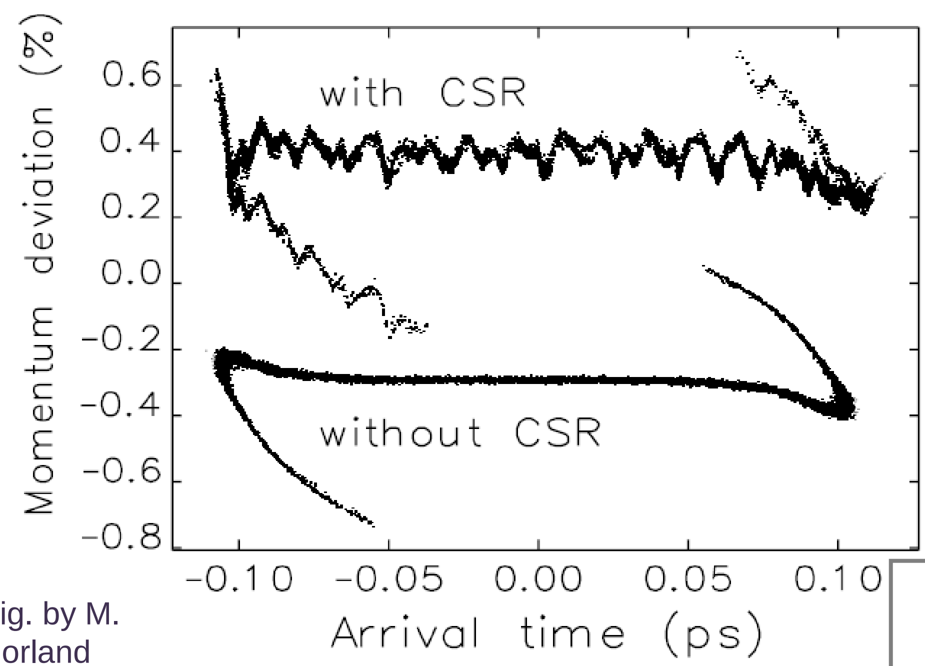


Fig. by M. Borland

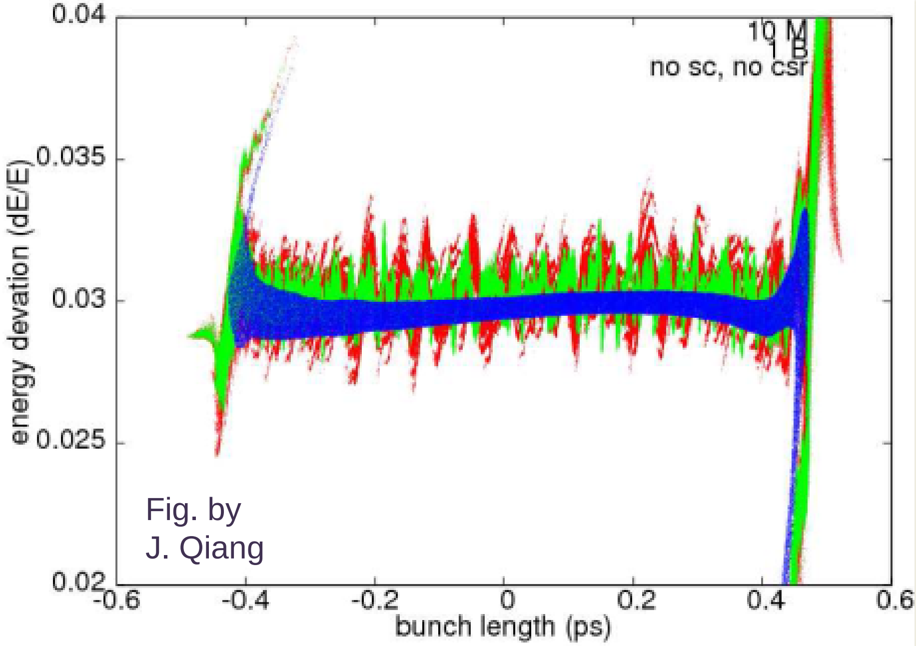


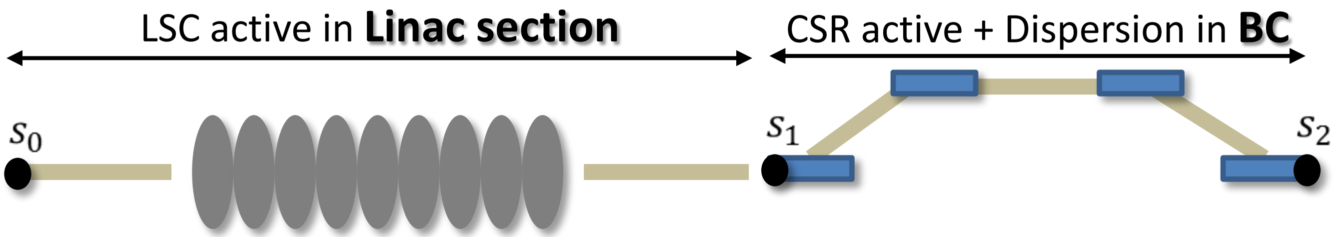
Fig. by J. Qiang

**Main adverse effect of micro-bunching instability is *growth in energy spread:***

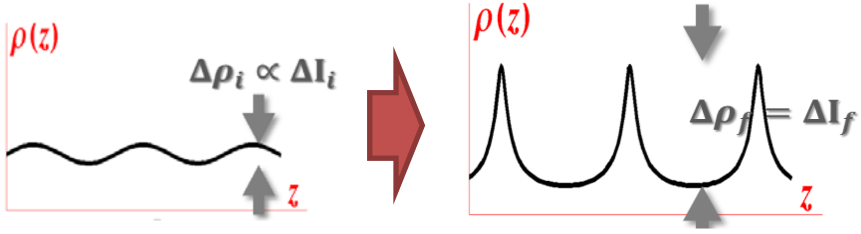
- *limits SASE performance,*
- *Degrades HG in seeding methods,*
- *reduces longitudinal coherence of radiation*

# Microbunching Gain, linear approximation

Courtesy of
M. Venturini



$$G = \frac{\Delta\rho_f/\rho_f}{\Delta\rho_i/\rho_i}$$



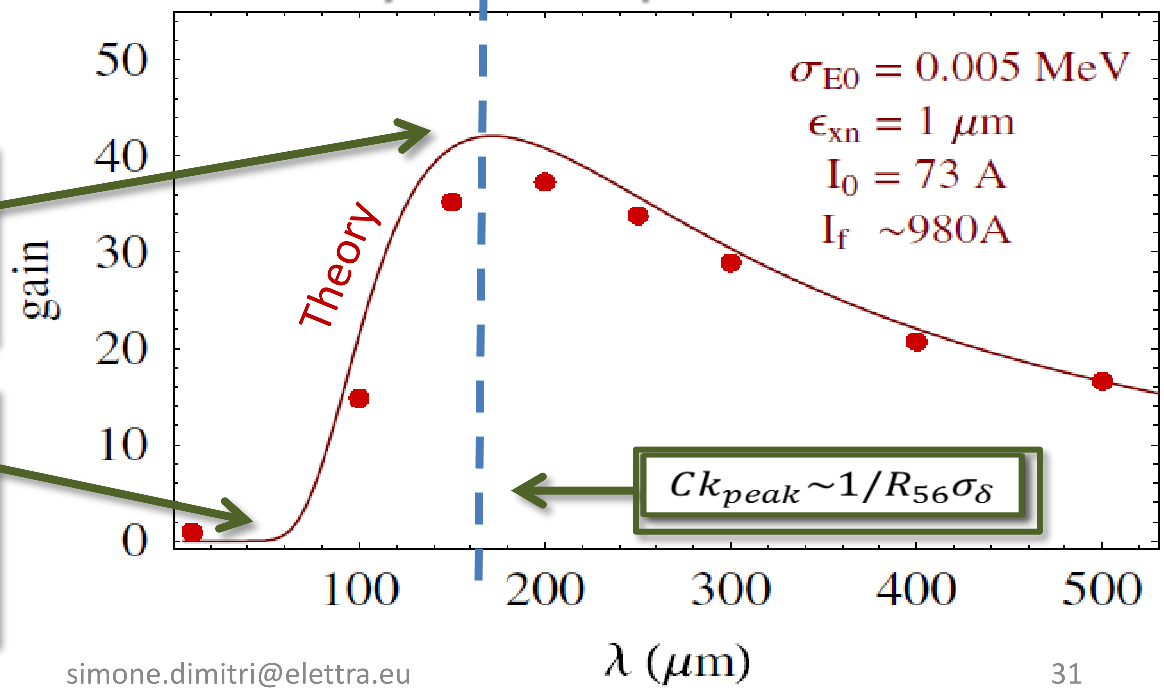
$$G \simeq 4\pi \frac{I_0}{I_A} L_s \frac{|Z(k)|}{Z_0\gamma_{BC}} (|R_{56}|Ck) e^{-(CkR_{56}\sigma_\delta)^2/2}$$

**Induces energy modulation through the linac**

**from dispersion in magnetic compressor**

**Gauss uncorrelated energy spread**

## Theory vs. macroparticle simulations



Final phase space appears modulated at wavelengths close to the maximum gain

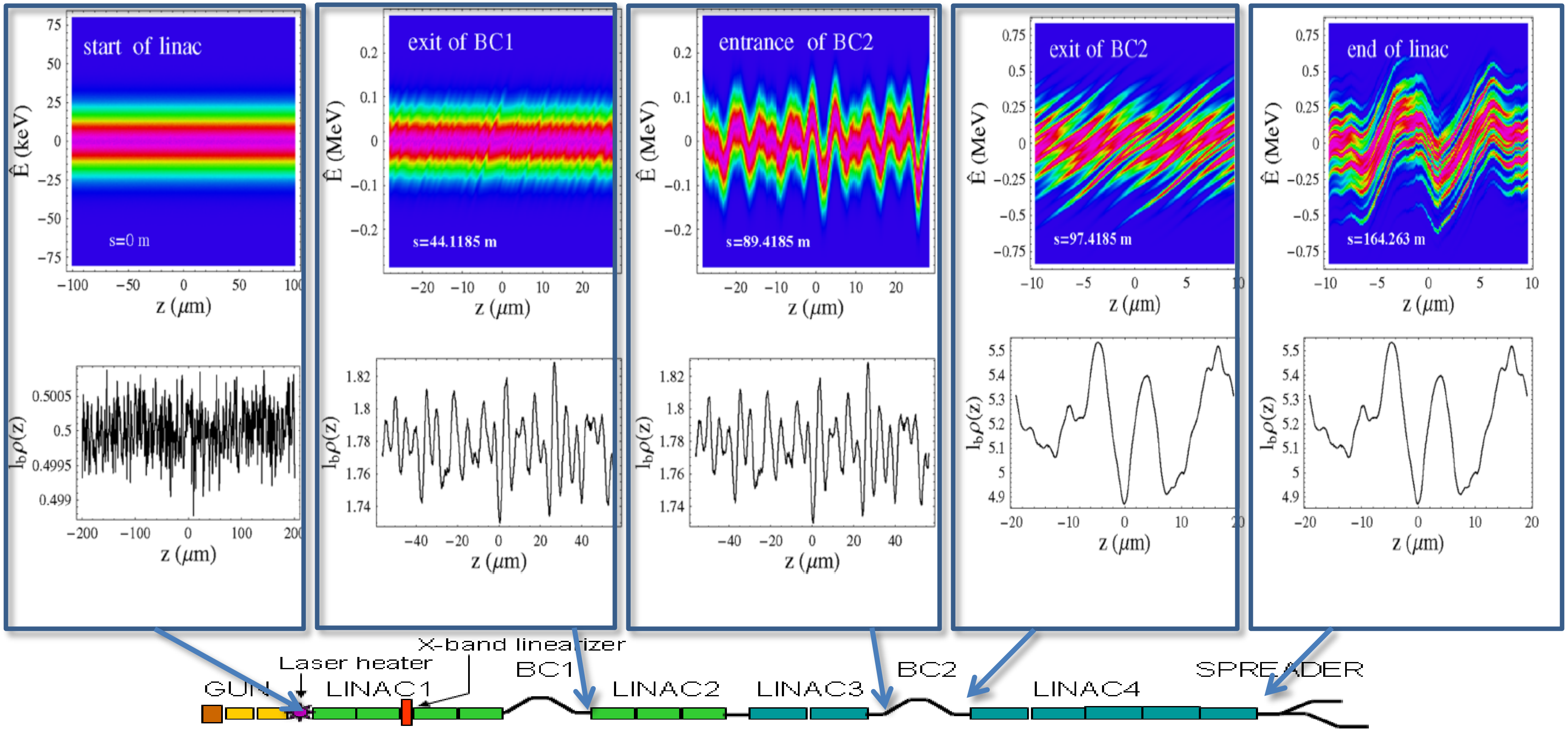
Gain is exp. Suppressed at short wavelengths, e.g. by large uncorrelated energy spread (longitudinal Landau damping)

# Microbunching Gain, *beyond* linear approximation

Courtesy of
M. Venturini

- Effect compounded by repeated compression through bunch compressors. In first approx.: $G_{tot} \simeq G_{BC1} \times G_{BC2} \times \cdots$
- If instability is large effects beyond the linear approximation used here can

**Study of $\mu B$-instability for FERMI:** *Longitudinal phase space, current profile at selected points*



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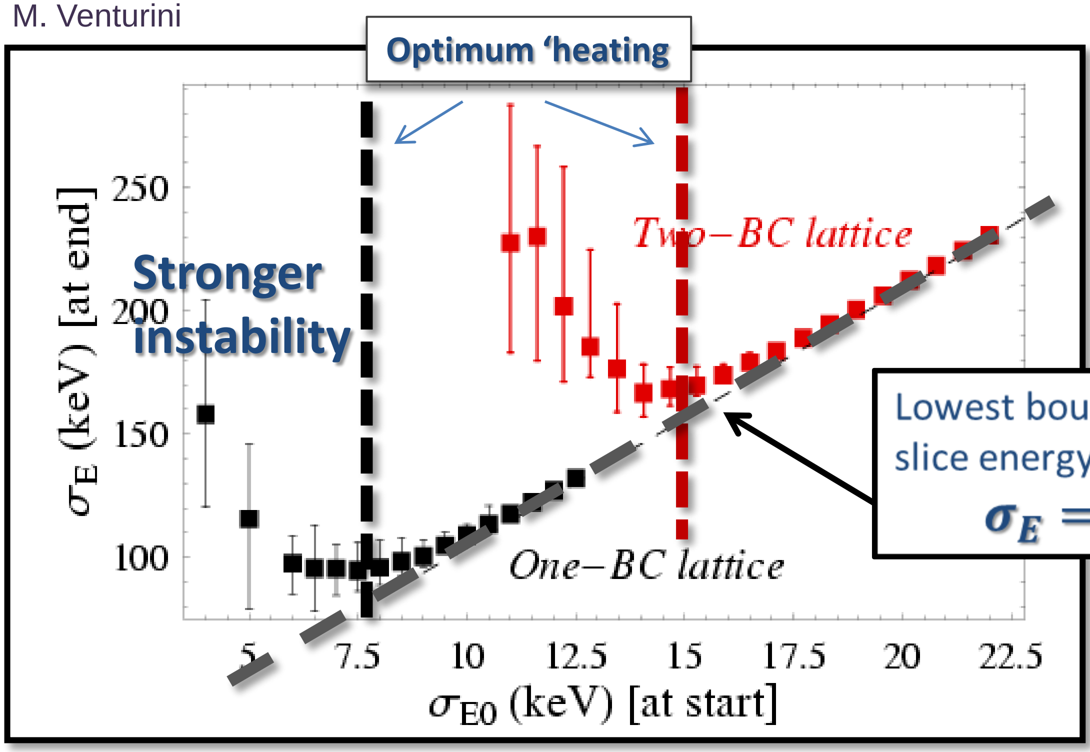
# Longitudinal Landau damping

- Through chicane, particles separated in energy by $\sigma_\delta$ move away from each other: $\boldsymbol{\Delta z = R_{56}\sigma_\delta}$ .
- This washes away clumps of charge (bunching) on the scale $\lambda$ if $\Delta z > \frac{\lambda}{2}$
- Leads to condition $\boldsymbol{CkR_{56}\sigma_\delta}$$\gtrsim$1 (exponential suppression in above Eq. ).

$$\boldsymbol{G \simeq 4\pi \frac{I_0}{I_A} L_s \frac{|Z(k)|}{Z_0\gamma_{BC}} (R_{56}Ck) e^{-(CkR_{56}\sigma_\delta)^2/2}}$$

Fig. by
M. Venturini



Lowest bound to final slice energy spread is

$$\boldsymbol{\sigma_E = C\sigma_{E0}}$$

There is an optimum of beam heating at low energy

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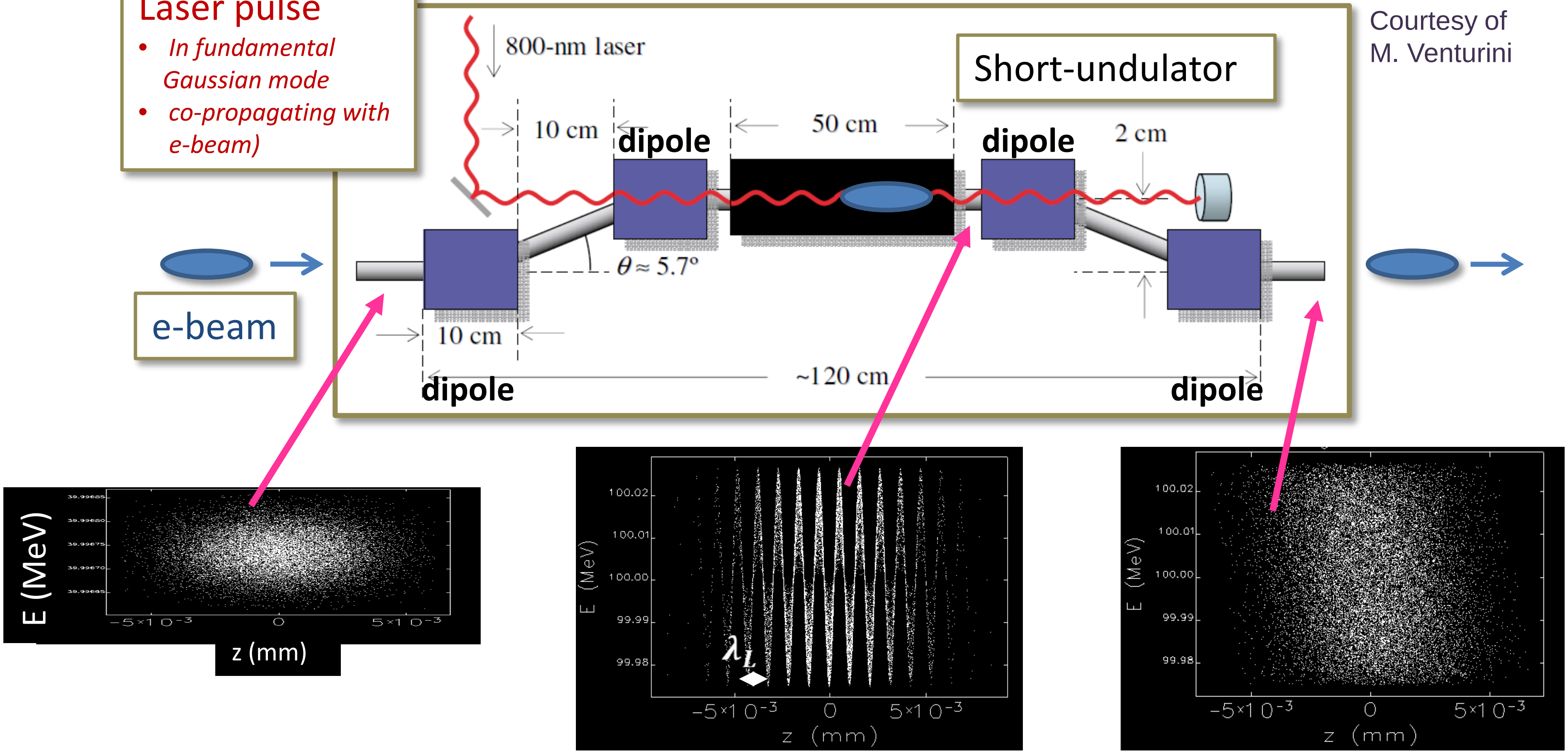
# Laser Heater

**Laser pulse**
- *In fundamental Gaussian mode*
- *co-propagating with e-beam)*



Short-undulator

e-beam

Courtesy of M. Venturini

$$P_L = 2P_0 \left(\frac{\sigma_E}{m_e c^2}\right)^2 (\sigma_x^2 + \sigma_r^2) \left(\frac{\gamma}{K[JJ]N_u \lambda_u}\right)^2$$

**Required laser pulse peak-power**

**e-beam rms size**

**Laser rms spot size**

$$z' = z + R_{51}x + R_{52}x' + R_{56}\delta$$

If angular spread is large the phase-space randomizes and energy spread becomes truly uncorrelated: $|R_{52}|\sigma_{x'} \gg \lambda_L/2\pi$

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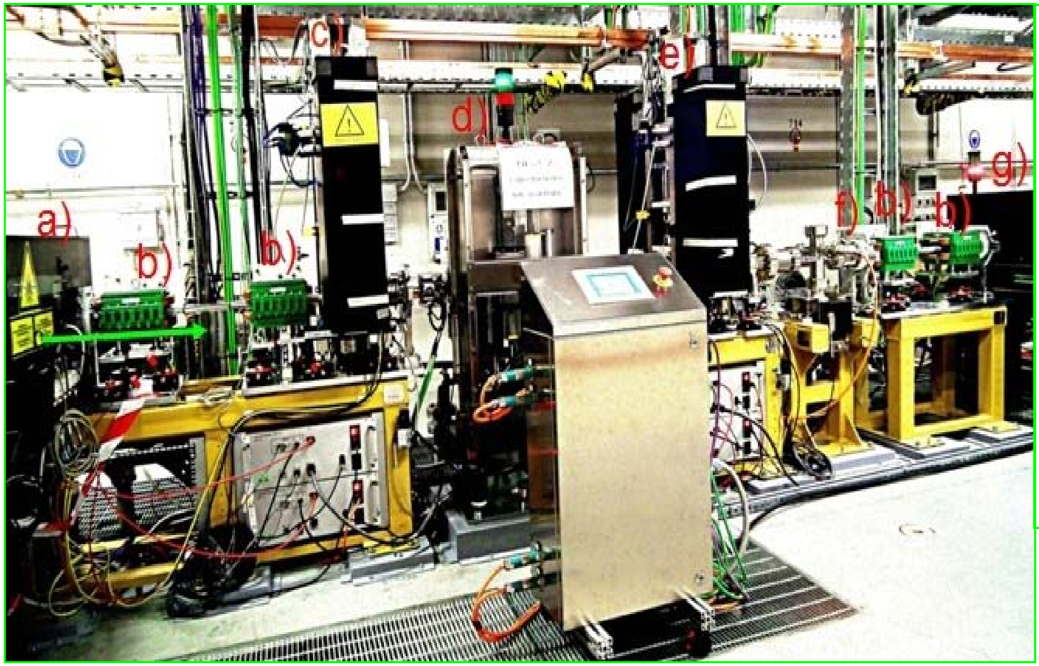
# Beam heating at FERMI

Data collected by S. Spampinati et al.



a) Input laser table (Ti:Sa, 783nm, 20ps)

b) Magnetic chicane for γ-injection (3.5°)

c) Cromox targets for e-/ γ spatial overlap

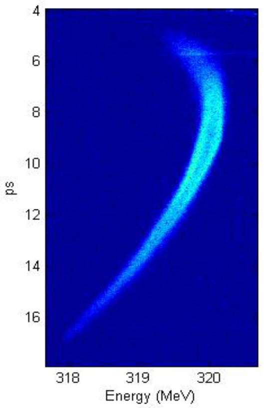
d) Undulator (L=0.4m, λu =40mm, K=0.9 )

e) BPM for e- trajectory feedback

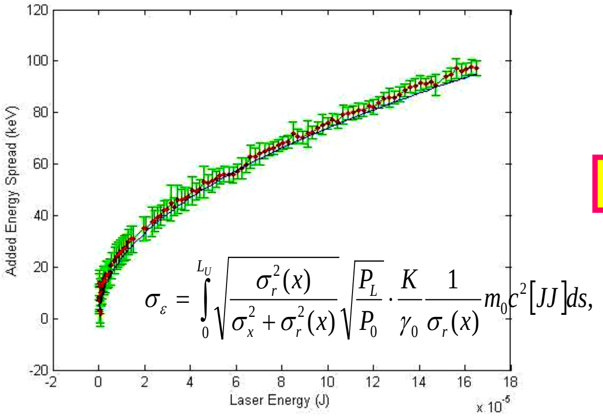
f) Laser delay line

g) Output laser table

☐ Beam heating:

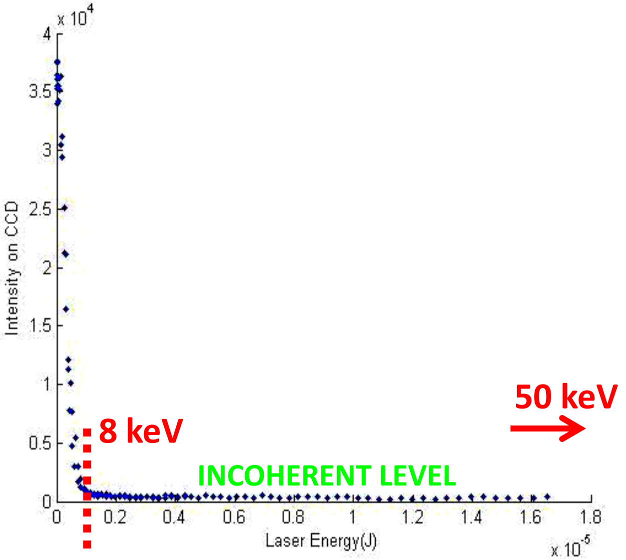


☐ Calibration, $\sigma_{E,LH}$ vs. $P_L$:



$$\sigma_\varepsilon = \int_0^{L_U} \sqrt{\frac{\sigma_r^2(x)}{\sigma_x^2+\sigma_r^2(x)}}\sqrt{\frac{P_L}{P_0}}\cdot\frac{K}{\gamma_0}\frac{1}{\sigma_r(x)}m_0c^2[JJ]ds,$$

☐ Suppression of μbi:



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# Acknowledgements / References (not exhaustive)

*Credits: M. Venturini (and myself), USPAS Course 2013, CO, USA.*

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***Thank You for Your kind attention***

**Questions and Comments are Welcome !**

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